JENN
Training and Consultancy
The path to enlightened education

Province of the
EASTERN CAPE
EDUCATION

SUBJECT: MATHEMATICS

GRADE 12

SPRING AND LAST PUSH

SOLUTIONS MANUAL

PAPER 1

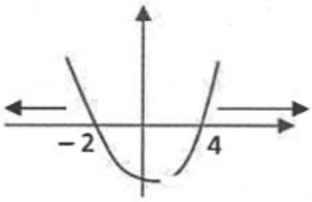
PAPER 2

Contents

PAPER 1

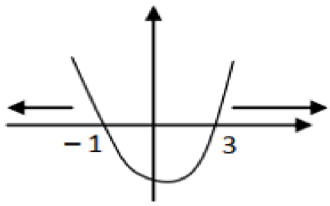
ALGEBRA

PAPER A

1.1.1	$3x^2+5x=0$ $x(3x+5)=0$ $x=0$ or $x=-\frac{5}{3}$
1.1.2	$4x^2+3x-5=0$ $x=\frac{-(3)\pm\sqrt{(3)^2-4(4)(-5)}}{2(4)}$ $x=0{,}80$ or $x=-1{,}55$
1.1.3	$(x-1)^2-9\geq 0$ $x^2-2x-8\geq 0$ $(x-4)(x+2)\geq 0$ $x=4$ or $x=-2$ $x\leq -2$ or $x\geq 4$ 
1.1.4	$5^{2x}-5^x=0$ $5^x(5^x-1)=0$ $5^x\neq 0$ or $5^x=1$ $x=0$ **OR/OF** $5^{2x}=5^x$ $2x=x$ $x=0$

1.1.5	$\frac{x}{\sqrt{20-x}}=1$ $x=\sqrt{20-x}$ $x^2=20-x$ $x^2+x-20=0$ $(x+5)(x-4)=0$ $x=4$ or $x\neq -5$
1.2	$2x^2-y^2=7$ .... (1) $x+y=9$ .... (2) $y=9-x$ $2x^2-(9-x)^2=7$ $2x^2-81+18x-x^2=7$ $x^2+18x-88=0$ $(x+22)(x-4)=0$ $x=-22$ or $x=4$ $y=31$ or $y=5$
	OR/*OF* $2x^2-y^2=7$ .... (1) $x+y=9$ .... (2) $x=9-y$ $2(9-y)^2-y^2=7$ $2(81-18y+y^2)-y^2-7=0$ $162-36y+2y^2-y^2-7=0$ $y^2-36y+155=0$ $(y-31)(y-5)=0$ $y=31$ or $y=5$ $x=-22$ or $x=4$
1.3	$P\times T=(1-a)(1+a)(1+a^2)(1+a^4)...(1+a^{512})$ $P\times T=(1-a^2)(1+a^2)(1+a^4)...(1+a^{512})$ $P\times T=(1-a^4)(1+a^4)...(1+a^{512})$ $P\times T=(1-a^8)...(1+a^{512})$ $P\times T=(1-a^{512})(1+a^{512})$ $=1-a^{1024}$

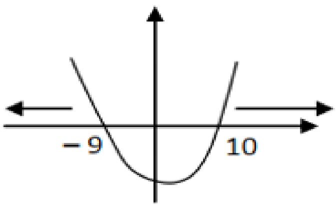
PAPER B

1.1.1	$x^2+x-12=0$ $(x-3)(x+4)=0$ $x=3$ or $x=-4$
1.1.2	$3x^2-2x=6$ $3x^2-2x-6=0$ $x=\dfrac{2\pm\sqrt{(-2)^2-4(3)(-6)}}{2(3)}$ $x=1{,}79$ or $x=-1{,}12$
1.1.3	$\sqrt{2x+1}=x-1$ $2x+1=(x-1)^2$ $2x+1=x^2-2x+1$ $x^2-4x=0$ $x(x-4)=0$ $x=0$ or $x=4$ $x\neq 0$ or $x=4$
1.1.4	$x^2-2x>3$ $x^2-2x-3>0$ $(x-3)(x+1)>0$ CV's: $x=-1$; $x=3$  $x<-1$ or $x>3$

1.2	$\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$ $x+2=2y \quad \ldots \quad (2)$ $x=2y-2$ $\frac{1}{2y-2}+\frac{1}{y}=1$ $y+2y-2=2y^2-2y$ $2y^2-5y+2=0$ $(2y-1)(y-2)=0$ $y=\frac{1}{2}$ or $y=2$ $x=-1$ or $x=2$
	OR/*OF* $\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$ $x+2=2y \quad \ldots \quad (2)$ $y=\frac{x}{2}+1$ $\frac{1}{x}+\frac{1}{\frac{x}{2}+1}=1$ $\frac{1}{x}+\frac{2}{x+2}=1$ $x+2+2x=x^2+2x$ $x^2-x-2=0$ $(x+1)(x-2)=0$ $x=-1$ or $x=2$ $y=\frac{1}{2}$ or $y=2$

1.3	$2^{m+1}+2^{m}=3^{n+2}-3^{n}$ $2^{m}(2+1)=3^{n}(3^{2}-1)$ $2^{m}(3)=3^{n}(8)$ $2^{m}(3)=3^{n}(2^{3})$ $\therefore m=3$ and $n=1$ $\therefore m+n=4$ **OR/*OF*** $2^{m+1}+2^{m}=3^{n+2}-3^{n}$ $2^{m}(2+1)=3^{n}(3^{2}-1)$ $2^{m}(3)=3^{n}(8)$ $2^{m}(3)=3^{n}(2^{3})$ $2^{m-3}=3^{n-1}$ Only true if $m-3=0$ and $n-1=0$ $\therefore m+n=4$

PAPER C

1.1.1	$(3x-6)(x+2)=0$ $x=2$ or $x=-2$
1.1.2	$2x^{2}-6x+1=0$ $x=\dfrac{-b\pm\sqrt{b^{2}-4ac}}{2a}$ $x=\dfrac{6\pm\sqrt{(-6)^{2}-4(2)(1)}}{2(2)}$ $x=2{,}82$ or $x=0{,}18$
1.1.3	$x^{2}-90>x$ $x^{2}-x-90>0$ $(x+9)(x-10)>0$ CV: $x=-9$ or $x=10$  $x<-9$ or $x>10$ **OR/OF** $(-\infty\,;-9)$ or $(10\,;\infty)$

1.1.4	$x-7\sqrt{x}=-12$ $x+12=7\sqrt{x}$ $(x+12)^2=(7\sqrt{x})^2$ $x^2+24x+144=49x$ $x^2-25x+144=0$ $(x-16)(x-9)=0$ $x=16$ or $x=9$
	OR/*OF* $x-7\sqrt{x}+12=0$ $(\sqrt{x}-3)(\sqrt{x}-4)=0$ or let $\sqrt{x}=k$ $\sqrt{x}=3$ or $\sqrt{x}=4$ $x=9$ or $x=16$

1.2	$2x-y=2$ $y=2x-2$ ....(1) $xy=4$ ....(2) (1) in (2): $x(2x-2)=4$ $2x^2-2x-4=0$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2$ or $x=-1$ $y=2$ $y=-4$

OR/*OF*

$2x - y = 2$

$x = \frac{1}{2}y + 1 \quad ....(1)$

$xy = 4 \quad ....(2)$

(1) in (2):

$y\left(\frac{1}{2}y + 1\right) = 4$

$\frac{1}{2}y^2 + y - 4 = 0$

$y^2 + 2y - 8 = 0$

$(y+4)(y-2) = 0$

$y = -4 \quad \text{or} \quad y = 2$

$x = -1 \qquad x = 2$

OR/OF

$2x - y = 2 \quad ....(1)$

$y = \frac{4}{x} \quad ....(2)$

(2) in (1):

$2x - \frac{4}{x} = 2$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad \text{or} \quad x = -1$

$y = 2 \qquad y = -4$

	OR/OF $2x - y = 2 \quad \ldots\ldots\ldots(1)$ $x = \frac{4}{y} \quad \ldots\ldots\ldots(2)$ (2)in (1): $2\left(\frac{4}{y}\right) - y = 2$ $8 - y^2 - 2y = 0$ $y^2 + 2y - 8 = 0$ $(y+4)(y-2) = 0$ $y = -4 \quad \text{or} \quad y = 2$ $x = -1 \qquad x = 2$
1.3	$2.5^n - 5^{n+1} + 5^{n+2} = 2.5^n - 5^n.5^1 + 5^n.5^2$ $= 5^n(2 - 5 + 25)$ $= 5^n(22)$ $2(5^n(11))$ **OR/OF** Any integer multiplied by an even number will be even
1.4	$\frac{3^{y+1}}{32} = \sqrt{96^x}$ $\frac{3^{y+1}}{2^5} = (96)^{\frac{x}{2}}$ $3^{y+1}.2^{-5} = 2^{\frac{5x}{2}}.3^{\frac{x}{2}}$ $-5 = \frac{5x}{2}$ $\therefore x = -2$ $y + 1 = \frac{x}{2}$ $y + 1 = \frac{-2}{2}$ $\therefore y = -2$

PAPER D

QUESTION 1

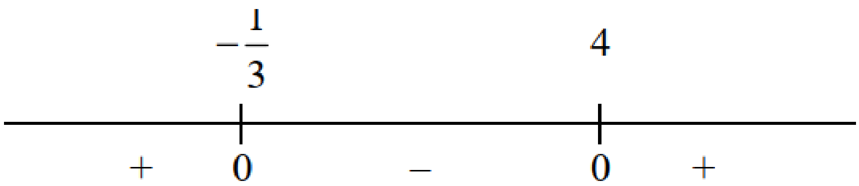
(a) (1) $x^2 - 5x = -6$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2 \quad \text{or} \quad x = 3$$

(2) $(3x+1)(x-4) < 0$



$$-\frac{1}{3} < x < 4$$

(3) $\log_2(x+6) = 1$

$$x + 6 = 2$$

$$x = -4$$

(4) $2x + \sqrt{x+1} = 1$

$$\sqrt{x+1} = 1 - 2x$$

$$x + 1 = 1 - 4x + 4x^2$$

$$4x^2 - 5x = 0$$

$$x(4x-5) = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{5}{4}$$

Check $x = 0$:

$$\text{LHS} = 2\times 0 + \sqrt{0+1}$$

$$= 1 = \text{RHS}$$

Check $x = \frac{5}{4}$

$$\text{LHS} = 2\left(\frac{5}{4}\right) = \sqrt{\frac{5}{4}+1}$$

$$= 4 \neq \text{RHS}$$

(5) $12^{5+3x} = 1$

$5 + 3x = 0$

$x = \dfrac{-5}{3}$

(b) $2x - y = 8$ .... ①

$x^2 - xy + y^2 = 19$ ...... ②

① : $y = 2x - 8$

② : $x^2 - x(2x-8) + (2x-8)^2 = 19$

$x^2 - 2x^2 + 8x + 4x^2 - 32x + 64 = 19$

$3x^2 - 24x + 45 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ or $x = 5$

$y = 2 \times 3 - 8$ or $y = 2 \times 5 - 8$

$= -2$ $\qquad = 2$

(c) $f(x) = x^{10} - 2x^5 + c$

$f(-1) = (-1)^{10} - 2(-1)^5 + c = 0$

$1 + 2 + c = 0$

$c = -3$

(d) $y = x^2$

$\dfrac{dy}{dx} = 2x$

At $(-1;\ 1)$: $m = 2\,(-1)$

$= -2$

PAPER E

QUESTION 1

(a) (1) $\dfrac{4x}{2} - \dfrac{2x+1}{3} = 5$

$\dfrac{12x - 4x - 2}{6} = \dfrac{30}{6}$ **OR** $12x - 2\,(2x + 1) = 30$

$8x = 32$

$x = 4$

(2) $(x-5)(x-6) \le 56$

$x^2 - 11x + 30 \le 56$

$x^2 - 11x - 26 \le 0$

$(x-13)(x+2) \le 0$

Critical Values: $13\ ;\ -2$

$-2 \le x \le 13$

(d) $c = -1$ or $c = -\frac{1}{4}$ (other answers possible)

(e) $3 - k < 0 \ \therefore k > 3$

PAPER F

(a) (1)

$$(x-1)^2 = 2(1-x)$$
$$(x-1)^2 = -2(x-1)$$
$$(x-1)^2 + 2(x-1) = 0$$
$$(x-1)(x-1+2) = 0$$
$$(x-1)(x+1) = 0$$
$$x = 1 \qquad x = -1$$

(2)

$$5^{-x}.5^{x-2} = \frac{25^{2x}}{5}$$
$$5^{-x}.5^{x-2} = \frac{5^{4x}}{5^1}$$
$$5^{-x+x-2} = 5^{4x-1}$$
$$-2 = 4x - 1$$
$$x = -\frac{1}{4}$$

PAPER G

1.1	1.1.1	$(2x-3)^2 = 1$ $2x - 3 = \pm 1$ $x = \frac{3 \pm 1}{2}$ $x = 1$ or $x = 2$ **OR** $4x^2 - 12x + 9 = 1$ $4x^2 - 12x + 8 = 0$ $4(x-1)(x-2) = 0$ $x = 1$ or $x = 2$

1.1.2

$$2x^2 + 4x - 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-7)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{72}}{4}$$

$$= \frac{-2 \pm 3\sqrt{2}}{2}$$

1.1.3

$$x - \sqrt{2x - 1} = 2$$

$$(x-2)^2 = \left(\sqrt{2x-1}\right)^2$$

$$x^2 - 4x + 4 = 2x - 1$$

$$x^2 - 6x + 5 = 0$$

$$(x-1)(x-5) = 0$$

$$x \neq 1 \quad \text{or} \quad x = 5$$

$$\therefore \; x = 5$$

1.2.1

$$x^2 + 5xy + 6y^2 = 0$$

$$(x+3y)(x+2y) = 0$$

$$x + 3y = 0 \qquad\qquad x + 2y = 0$$

$$x = -3y \qquad \text{OR} \qquad x = -2y$$

$$\frac{x}{y} = -3 \qquad\qquad \frac{x}{y} = -2$$

OR

$$x^2 + 5xy + 6y^2 = 0$$

$$x = \frac{-5y \pm \sqrt{(5y)^2 - 4(1)(6y^2)}}{2(1)}$$

$$x = \frac{-5y \pm \sqrt{y^2}}{2}$$

$$x = \frac{-5y \pm y}{2}$$

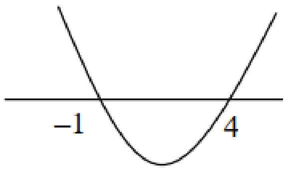
$$x = -3y \qquad x = -2y$$

$$\frac{x}{y} = -3 \quad \text{or} \quad \frac{x}{y} = -2$$

1.2.2	$x+y=8$ $-3y+y=8$ $-2y=8$ $y=-4$ $x=12$ OR $x+y=8$ $-2y+y=8$ $-y=8$ $y=-8$ $x=16$

1.3.1	$\sqrt{2p+5}=0$ $2p+5=0$ $2p=-5$ $p=-\frac{5}{2}$
1.3.2	$2p+5<0$ $p<-\frac{5}{2}$

PAPER H

1.1.4	$(x+1)(4-x)>0$ $(x+1)(x-4)<0$ $+ \quad 0 \quad - \quad 0 \quad +$ $\quad -1 \qquad 4$ or  $-1<x<4$

1.2.1	$2^x+2^{x+2}=-5y+20$ $2^x(1+2^2)=-5y+20$ $2^x=\frac{-5y+20}{5}$

1.2.2	If $y=-4$, $2^x+2^{x+2}=-5y+20$ $2^x+2^{x+2}=40$ $2^x\left(1+2^2\right)=40$ $2^x=8$ $2^x=2^3$ $x=3$

1.2.3	$-y+4>0$ $y<4$ Largest integer value of y is 3 $2^x=-3+4$ $2^x=1$ $x=0$

PAPER I

QUESTION 1

1.1.1 $(x-3)(x+4)=18$

$\therefore x^2+x-12=18$ ✓

$\therefore x^2+x-30=0$ ✓

$\therefore (x+6)(x-5)=0$ ✓

$\therefore x=-6$ or $x=5$ ✓

1.1.2 $x^2=6(x+2)$

$\therefore x^2=6x+12$ ✓

$\therefore x^2-6x-12=0$ ✓

$$\therefore x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(-12)}}{2(1)}$$ ✓

$$\therefore x=\frac{6\pm\sqrt{84}}{2}$$

$\therefore x=7,58$ ✓ or $x=-1,58$ ✓

1.1.3 $2-\frac{1}{x}=\frac{3}{x+2}$

$\therefore 2x(x+2)-(x+2)=3x$ ✓

$\therefore 2x^2+4x-x-2=3x$ ✓✓

$\therefore 2x^2-2=0$ ✓

$\therefore x^2-1=0$

$\therefore (x+1)(x-1)=0$ ✓

$\therefore x=-1$ or $x=1$ ✓

1.2 $y^2 + 2y - \dfrac{8}{y^2 + 2y} = 7$

Let $y^2 + 2y = k$

$\therefore k - \dfrac{8}{k} = 7$ ✓

$\therefore k^2 - 7k - 8 = 0$ ✓

$\therefore (k+1)(k-8) = 0$ ✓

$\therefore k = -1$ or $k = 8$ ✓

$\therefore y^2 + 2y = -1$ or $y^2 + 2y = 8$

$\therefore y^2 + 2y + 1 = 0$ or $y^2 + 2y - 8 = 0$ ✓

$\therefore (y+1)(y+1) = 0$ ✓ or $(y+4)(y-2) = 0$ ✓

$\therefore y = -1$ or $y = -4$ or $y = 2$ ✓

1.3 $3x^2 - 6px - 9p^2 = 0$

$\therefore x^2 - 2px - 3p^2 = 0$ ✓

$\therefore x^2 - 2px = 3p^2$

$\therefore x^2 - 2px + p^2 = 3p^2 + p^2$ ✓

$\therefore (x-p)^2 = 4p^2$ ✓

$\therefore x - p = \pm\sqrt{4p^2}$ ✓

$\therefore x = p \pm \sqrt{4p^2}$

$\therefore x = p + 2p$ or $x = p - 2p$ ✓

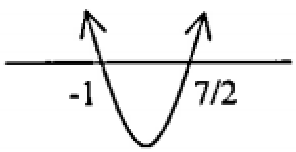
$\therefore x = 3p$ or $x = -p$ ✓

1.4 $2x^2 - 5x \geq 7$

$\therefore 2x^2 - 5x - 7 \geq 0$ ✓

$\therefore (x+1)(2x-7) \geq 0$ ✓

$\therefore x \leq -1$ ✓ or $x \geq \dfrac{7}{2}$ ✓



QUESTION 2

2.1 $(2y+3)(x^2+4)=0$

$\therefore y=-\frac{3}{2}$ ✓

or $x^2=-4$ ✓

$\therefore$ no real value for x ✓

2.2 $2a-b=7$

$\therefore b=2a-7$ ✓

Subs $b=2a-7$ into $a^2+ab+b^2=7$

$a^2+a(2a-7)+(2a-7)^2=7$ ✓

$\therefore a^2+2a^2-7a+4a^2-28a+49=7$ ✓✓

$\therefore 7a^2-35a+42=0$ ✓

$\therefore a^2-5a+6=0$

$\therefore (a-2)(a-3)=0$ ✓

$\therefore a=2$ or $a=3$ ✓

$\therefore b=-3$ or $b=-1$ ✓

2.3 $xy=20$ ✓

$\therefore y=\frac{20}{x}$

$(x+3)(y+1)=40$ ✓

$\therefore xy+x+3y+3=40$

Subs $y=\frac{20}{x}$ into $xy+x+3y+3=40$

$\therefore x\left(\frac{20}{x}\right)+x+3\left(\frac{20}{x}\right)+3=40$

$\therefore 20+x+\frac{60}{x}+3=40$

$\therefore 20x+x^2+60+3x-40x=0$

$\therefore x^2-17x+60=0$ ✓

$\therefore (x-5)(x-12)=0$

$\therefore x=5$ or $x=12$ ✓

$\therefore y=4$ or $y=\frac{5}{3}$ ✓

Dimensions $\therefore 5\times 4$ or $12\times\frac{5}{3}$

PATTERNS, SEQUENCES AND SERIES

PAPER A

2.1.1	$r = \frac{1}{2}$ Yes, because $-1 < \frac{1}{2} < 1$
2.1.2	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{4}{1-\frac{1}{2}}$ $\therefore S_\infty = 8$
2.2	$\sum_{p=k}^{10} 3^{p-1} = 3^{k-1} + 3^{k+1-1} + 3^{k+2-1} + \ldots + 3^9$ $= 3^{k-1} + 3^k + 3^{k+1} + \ldots 3^9$ $S_n = \frac{a(r^n - 1)}{r-1}$ $29\,520 = \frac{3^{k-1}(3^{11-k} - 1)}{3-1}$ $3^{10} - 3^{k-1} = 59\,040$ $3^{k-1} = 9$ $k - 1 = 2$ $\therefore k = 3$

3.1.1	$3 ; 7 ; 12 ; 18$ $\vee \ \vee \ \vee$ First diff: $4 ; 5 ; 6$ $\vee \ \vee$ Second diff: $1 ; 1$ $2a = 1$ $a = \frac{1}{2}$ $3a + b = 4$ $3\left(\frac{1}{2}\right) + b = 4$ $b = \frac{5}{2}$ $a + b + c = 3$ $\frac{1}{2} + \frac{5}{2} + c = 3$ $c = 0$

	$T_n = \frac{1}{2}n^2 + \frac{5}{2}n$
3.1.2	$13\,527 = \frac{1}{2}n^2 + \frac{5}{2}n$ $n^2 + 5n - 27\,054 = 0$ $(n-162)(n+167) = 0$ $n = 162$ or $n = -167$ $T_{161} = 13\,363$ $13\,527 - 13363 = 164$ 164 must be added.
3.2.1	$T_n = 8 + (n-1)(3)$ $T_n = 3n + 5$ $41 = 3n + 5$ $36 = 3n$ $n = 12$
3.2.2a	$P_{41} = 12$
3.2.2b	$P_8 = a + 7d = 1$ $P_{11} = a + 10d = 2$ $3d = 1$ $d = \frac{1}{3}$ $a + 7\left(\frac{1}{3}\right) = 1$ $a = -\frac{4}{3}$ **OR/*OF*** $n = 3P_n + 5$ $1 = 3P_1 + 5$ $-4 = 3P_1$ $P_1 = -\frac{4}{3}$ **OR/*OF*** $T_n = 3n + 5$ $1 = 3n + 5$ $n = -\frac{4}{3}$ $\therefore P_1 = -\frac{4}{3}$

PAPER B

2.1.1	$7+12+17+\ldots\ldots$ $T_n = a+(n-1)d$ $T_{91} = 7+(91-1)(5)$ $T_{91} = 457$ **OR/*OF*** $d=5$ $T_n = 5n+2$ $T_{91} = 5(91)+2$ $T_{91} = 457$
2.1.2	$S_n = \frac{n}{2}[2a+(n-1)d]$ $S_{91} = \frac{91}{2}[2\times 7+(91-1)(5)]$ $S_9 = 21\ 112$ **OR/*OF*** $S_n = \frac{n}{2}(a+l)$ $S_{91} = \frac{91}{2}(7+457)$ $S_{91} = 21\ 112$
2.1.3	$T_n = 7+(n-1)(5)$ $5n+2=517$ $5n=515$ $n=103$

2.2.1	$T_1 = 3\,;\, T_2 - T_1 = 9$ and $T_3 - T_2 = 21$ $\begin{array}{ccccccccc} 3 & & 12 & & 33 & & 66 & & 111 \\ & 9 & & 21 & & 33 & & 45 & \\ & & 12 & & 12 & & 12 & & \end{array}$ $\therefore T_5 = 3+9+21+33+45 = 111$ **OR/*OF*** $2a=12$ $a=6$ $3(6)+b=9$ $b=-9$ $6-9+c=3$ $T_5 = 6(5)^2-9(5)+6 = 111$

2.2.2	$2a=12$ $a=6$ $3(6)+b=9$ or $5\times 6+b=21$ $b=-9$ $6-9+c=3$ $c=6$ $T_n=6n^2-9n+6$
2.2.3	$T_n{}'=12n-9>0$ $n>\frac{3}{4}$ $\therefore$ T_n is increasing for $n\in N$ **OR/*OF*** $n=-\frac{b}{2a}=-\frac{-9}{2(6)}$ $n=\frac{3}{4}$ $\therefore$ min at $n=1$ for $n\in N$ $\therefore$ T_n is increasing for $n\in N$

3.1.1	$T_n=ar^{n-1}$ $T_n=3(2)^{n-1}$
3.1.2	$\sum_{p=1}^{k}\frac{3}{2}.2^p=98\ 301$ $\sum_{p=1}^{k}\frac{3}{2}.2^p=3+6+12+\ldots$ $n=k$ $\frac{3\left[(2)^k-1\right]}{2-1}=98\ 301$ $(2)^k=32\ 768$ $2^k=2^{15}$ **OR/*OF*** $k=\log_2 32\ 768$ $\therefore k=15$

3.2	$S_{22}=\frac{22}{2}\left[2a+21(3)\right]$ $S_{22}=22a+693$ $S_{\infty}=\frac{a}{1-\frac{1}{3}}$ $=\frac{3a}{2}$ $\therefore 22a+693=\frac{3a}{2}+734$ $44a+1386=3a+1468$ $41a=82$ $a=2$

PAPER C

2.1.1	$a=14$ $T_6=14r^5=448$ $r^5=32$ $\therefore r=2$ Answer only: full marks
2.1.2	$T_n=14(2)^{n-1}$ $S_n=\frac{14(2^6-1)}{2-1}$ $S_6=882$ $114\ 674-882=113\ 792$ $113\ 792=896\left(2^n-1\right)$ $128=2^n$ $n=7$ **OR/OF** $S_n=\frac{a(r^n-1)}{r-1}$ $114\ 674=\frac{14(2^n-1)}{2-1}$ $8\ 191=2^n-1$ $2^n=8\ 192$ $n=\log_2 8\ 192$ $n=13$ $\therefore$ 7 more terms must be added to the first 6 terms.

2.1.3	$r = \frac{1}{2}$ OR $448r^5 = 14$ $\therefore r = \frac{1}{2}$ $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{448}{1-\frac{1}{2}}$ $S_\infty = 896$
2.2	$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right) = 20\frac{1}{6}$ $T_1 = \frac{1}{6}$ $T_2 = \frac{1}{3}+\frac{1}{6} = \frac{3}{6}$ $d = \frac{3}{6}-\frac{1}{6} = \frac{1}{3}$ $\frac{121}{6} = \frac{n}{2}\left[2\left(\frac{1}{6}\right)+(n-1)\left(\frac{1}{3}\right)\right]$ $\frac{121}{3} = n\left[\frac{1}{3}+\frac{1}{3}n-\frac{1}{3}\right]$ $\frac{121}{3} = \frac{1}{3}n^2$ $121 = n^2$ $n = 11$ $\therefore k = 10$
	OR/*OF* $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right) = 20\frac{1}{6}$ $a = \frac{1}{6}$ $l = \frac{1}{3}k+\frac{1}{6}$ $n = k+1$
	$S_n = \frac{n}{2}[a+l]$ $\frac{121}{6} = \frac{k+1}{2}\left[\frac{1}{6}+\frac{1}{3}k+\frac{1}{6}\right]$ $\frac{121}{6} = \frac{k+1}{2}\left[\frac{1}{3}k+\frac{1}{3}\right]$

	$\frac{121}{6}=\frac{k+1}{2}\left[\frac{k+1}{3}\right]$ $\frac{121}{6}=\frac{(k+1)^2}{6}$ $k+1=\pm\sqrt{121}$ $k+1=11$ $k=10$
3.1	$3a+b=7$ $3+b=7$ $b=4$ **OR/OF** $T_2-T_1=7$ $4+2b+9-(1+b+9)=7$ $b=4$
3.2	$T_n=n^2+4n+9$ $T_{60}=(60)^2+4(60)+9$ $=3849$ Answer only: full marks
3.3	14 ; 21 ; 30 ; 41; First difference: 7 ; 9 ; 11 ; … Common 2nd difference: 2 $T_p=2p+5$ Answer only: full marks **OR/*OF*** First difference: 7 ; 9 ; 11 ; … $T_n=a+(n-1)d$ $T_p=7+(p-1)(2)$ $T_p=2p+5$
3.4	$157=2p+5$ $p=76$ $\therefore$ Between T_{76} and T_{77}
	$T_{n+1}-T_n=157$ $(n+1)^2+4(n+1)+9-(n^2+4n+9)=157$ $n^2+2n+1+4n+4+9-n^2-4n-9=157$ $2n=152$ $n=76$ $\therefore$ Between T_{76} and T_{77}

PAPER D

4.1 The second, third, fourth and fifth terms are 1 ; – 6 ; T_4 and – 14

First differences are: – 7 ; $T_4 + 6$; – 14 – T_4

So $T_4 + 6 + 7 = -14 - 2T_4 - 6$

$T_4 = -11$

d = – 11 + 6 + 7 = 2 or – 14 + 22 – 6 = 2

OR

T_2		T_3		T_4		T_5
1		-6				-14
	-7		$-7+d$		$-7+2d$	
		d		d		

$$T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$$
$$-15 = (-7 + 2d) + (-7 + d) + -7$$
$$-15 = -21 + 3d$$
$$6 = 3d$$
$$d = 2$$

OR

$$4a + 2b + c = 1$$
$$9a + 3b + c = -6$$
$$5a + b = -7$$

$$25a + 5b + c = -14$$
$$16a + 2b = -8$$
$$10a + 2b = -14$$
$$6a = 6$$
$$a = 1$$
$$d = 2a = 2$$

4.2

$$\begin{array}{ccccccccc} T_1 & & & & 1 & & & & -6 \\ & & -9 & & & & -7 & & \\ & & & & 2 & & & & \end{array}$$

$T_1 = 10$

OR

$$a = 1$$
$$5a + b = -7$$
$$5(1) + b = -7$$
$$b = -12$$
$$a + b + c = 1$$
$$4(1) + 2(-12) + c = 1$$
$$c = 21$$
$$T_n = n^2 - 12n + 21$$
$$T_1 = (1)^2 - 12(1) + 21$$
$$= 10$$

QUESTION/VRAAG 7

7.1 29

7.2

$$T_n = an^2 + bn + c$$
$$1 = a + b + c$$
$$\therefore c = 1 - a - b$$
$$5 = 4a + 2b + c$$
$$5 = 4a + 2b + 1 - a - b$$
$$4 = 3a + b \qquad (1)$$
$$11 = 9a + 3b + c$$
$$11 = 9a + 3b + 1 - a - b$$
$$\therefore 10 = 8a + 2b \qquad (2)$$

Solving (1) and (2) simultaneously.
Los (1) en (2) gelyktydig op.

$8 = 6a + 2b$ (1) x 2
$\underline{10 = 8a + 2b \quad (2)}$

$$\therefore 2 = 2a$$
$$\therefore a = 1$$
$$\therefore b = 1$$
$$\therefore c = -1$$
$$T_n = n^2 + n - 1$$

Or/Of

1 5 11 19 29

4 6 8 10

2 2 2

$2a = 2$	$3a + b = 4$	$a + b + c = 1$
$a = 1$	$3 + b = 4$	$1 + 1 + c = 1$
	$b = 1$	$c = -1$

$T_n = n^2 + n - 1$

7.3	$T_n = n^2 + n - 1$ or/of $\therefore T_{100} = 100^2 + 100 - 1$ $= 10\,099$	$T_n = 100(101) - 1$ $= 10\,099$

PAPER E

QUESTION 4

4.1	$-7 \quad 0 \quad 9 \quad 20$ $7 \quad 9 \quad 11$ $2 \quad 2$ $2a = 2$ $a = 1$ $3(1) + b = 7$ $b = 4$ $(1) + (4) + c = -7$ $c = -12$ $\therefore T_n = n^2 + 4n - 12$ **OR** $2a = 2$ $a = 1$ $T_2 = 2^2 + b(2) + c = 0$ $2b + c = -4 \quad (1)$ $T_3 = 3^2 + b(3) + c = 9$ $3b + c = 0 \quad (2)$ OR $3(1) + b = 7$ $b = 4$ $1 + a + c = -7$ $c = -12$
	$(2) - (1) \quad b = 4$ $\therefore c = -4 - 2(4) = -12$ $T_n = n^2 + 4n - 12$

4.2	$n^2 + 4n - 12 = 128$ $n^2 + 4n - 140 = 0$ $(n+14)(n-10) = 0$ $n \neq -14$ or $n = 10$ invalid $\therefore n = 10$

4.3	$-7 ; 0 ; 9 ; 20 ; \ldots$ first difference7 9 11 second difference2 2 $F_n = 2n + c$ $F_1 = 2(1) + c = 7$ $\therefore c = 5$ $F_n = 2n + 5$
4.4	$F_n = 2n + 5 = 599$ $2n = 594$ $\therefore n = 297$ this difference will be between term 297 and term 298

QUESTION 5

5.1

Pattern	1	2	3
White squares	4	12	24

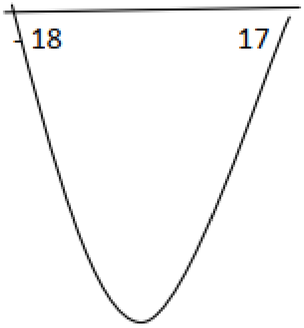
40

5.2

$$W_n = 2n^2 + 2n$$

$$W_{157} = 2(157)^2 + 2(157)$$

$$= 49612$$

5.3	$2n^2 + 2n + 1 < 613$ $2n^2 + 2n - 612 < 0$ $n^2 + n - 306 < 0$ $(n-17)(n+18) < 0$  $\therefore n = 16$

5.4	$P_n = 4n^2 + 4n + 1$ $= (2n)^2 + 2(2n) + 1$ $2n$ is even for all $n \in Z$ $\therefore$ Total squares used in the n^{th} pattern is always odd. **OR** $P_n = 4n^2 + 4n + 1$ $= 2(2n^2 + 2n) + 1$ $2(2n^2 + 2\text{n})$ is odd for all $n \in Z$ $2(2n^2 + 2n) + 1$ is odd for all $n \in \text{Z}$ $\therefore$ Total squares used in the n^{th} pattern is always odd.

PAPER F

QUESTION 3

3.1	$T_2 - T_1 = T_3 - T_2$ $p + 5 - 2p + 3 = 2p + 7 - p - 5$ $-p + 8 = p + 2$ $p = 3$
3.2	Pattern is $3\ ;\ 8\ ;\ 13\ ;\ \ldots$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{120} = \frac{120}{2}[2(3) + 119(5)]$ $= 36060$
3.3.1	$x = k + 1$ and $y = k + 2$

3.3.2	$T_x = a + (x-1)d = 3 + 5k$ $T_y = a + (k+1)d$ $= 3 + (k+1)(5)$ $= 8 + 5k$ $T_x + T_y = 11 + 10k$

QUESTION 4

4.1.1	15 ; 5
4.1.2	$S_\infty = \frac{a}{1-r}$ $= \frac{15}{1-\frac{1}{3}}$ $= \frac{45}{2} = 22{,}5$

4.2	$\sin 30° ; \cos 30° ; \frac{3}{2}$ $\frac{1}{2} ; \frac{\sqrt{3}}{2} ; \frac{3}{2}$ $a = \frac{1}{2} ; r = \sqrt{3}$ $ar^{n-1} = 40{,}5\sqrt{3}$ $\frac{1}{2}(\sqrt{3})^{n-1} = \frac{81}{2}\sqrt{3}$ $3^{\frac{n-1}{2}} = 3^4 . 3^{\frac{1}{2}}$ $\frac{n-1}{2} = 4\frac{1}{2} = \frac{9}{2}$ $n - 1 = 9$ $n = 10$

OR

	$\sin 30° ; \cos 30° ; \frac{3}{2}$ $\frac{1}{2} ; \frac{\sqrt{3}}{2} ; \frac{3}{2}$ $a = \frac{1}{2} ; r = \sqrt{3}$ $ar^{n-1} = \frac{81}{2}\sqrt{3}$ $\frac{1}{2}(\sqrt{3})^{n-1} = \frac{81}{2}\sqrt{3}$ $\frac{(\sqrt{3})^n}{\sqrt{3}} = 81\sqrt{3}$ $243 = (\sqrt{3})^n$ $3^5 = 3^{\frac{1}{2}n}$ $n = 10$

PAPER G

QUESTION 3

3.1

$$r=\frac{2(3x-1)^2}{2(3x-1)}$$
$$r=3x-1$$
$$-1<3x-1<1$$
$$0<3x<2$$
$$0<x<\frac{2}{3}$$

3.2

$$T_2=ar$$
$$6=kr$$
$$r=\frac{6}{k}\quad \ldots\quad (1)$$

$$S_\infty=25$$
$$S_\infty=\frac{a}{1-r}$$
$$25=\frac{k}{1-r}\quad \ldots\quad (2)$$

sub. (1) into (2)

$$25=\frac{k}{1-\frac{6}{k}}$$
$$k=25\left(1-\frac{6}{k}\right)$$
$$k=25-\frac{150}{k}$$
$$0=k^2-25k+150$$
$$0=(k-10)(k-15)$$
$$\therefore k=10 \quad \textbf{or} \quad \mathrm{k}=15$$

3.3

$$(1\times 2)+(5\times 6)+(9\times 10)+(13\times 14)+\ldots+(81\times 82)$$

$$(4n-3)\times(4n-2)$$
$$=16n^2-20n+6$$

$$4n-3=81 \qquad \textbf{OR} \qquad 4n-2=82$$
$$4n=84 \qquad\qquad 4n=84$$
$$n=21 \qquad\qquad n=21$$

$$\sum_{n=1}^{21}16n^2-20n+6 \qquad \textbf{OR} \qquad \sum_{n=1}^{21}(4n-3)(4n-2)$$

PAPER H

QUESTION 2

2.1.1	$\frac{1}{16}$; 13	
2.1.2	$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots to\ 25\ terms\right)$	$(4+7+10+13+\ldots to\ 25\ terms)$
	$\frac{a(r^n-1)}{r-1}$	$=\frac{n}{2}[2a+(n-1)d]$
	$=\frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}$	$=\frac{25}{2}[2(4)+24(3)]$
	$= 0{,}9999999$	$= 1\ 000$
	$S_{50}=1001{,}00$	

OR

S_{50} = 25 terms of 1st sequence + 25 terms of 2nd sequence

$$S_{50}=\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots to\ 25\ terms\right)+(4+7+10+13+\ldots to\ 25\ terms)$$

$$S_{50}=\frac{\frac{1}{2}\left(\left(\frac{1}{2}\right)^{25}-1\right)}{\frac{1}{2}-1}+\frac{25}{2}[2(4)+24(3)]$$

$$S_{50}=0{,}999999\ldots+1000$$

$$S_{50}=1001{,}00$$

2.2.1 60 ; 78

2.2.2

8   18   30   44

10   12   14

2   2

$2a = 2$

$a = 1$

$T_n = n^2 + bn + c$

$8 = 1 + b + c$

$7 = b + c \qquad \ldots\text{(i)}$

$18 = 4 + 2b + c$

$14 = 2b + c \qquad \ldots\text{(ii)}$

(ii) – (i): $\quad 14 = 2b + c$

$7 = b + c$

$\therefore\ 7 = b$

$c = 0$

$T_n = n^2 + 7n$

OR

$T_1 = 8$

$T_2 - T_1 = 10$

$T_3 - T_2 = 12$

$T_n - T_{n-1} = n$th term of sequence with $a = 8$ and $d = 2$

Add both sides

$T_n = 8 + 10 + 12 + \ldots +$ to 25 terms

$T_n = \frac{n}{2}[16 + 2(n-1)]$

$T_n = n(n+7)$

2.2.3

$n(n+7) = 330$

$n^2 + 7n - 330 = 0$

$(n+22)(n-15) = 0$

$n = -22 \quad \text{or} \quad n = 15$

$n = 15$

$\therefore 15^{\text{th}}$ term is 330.

2.3	$S_n = a + [a+d] + [a+2d] + \ldots + [a+(n-2)d] + [a+(n-1)d]$ $S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots + [a+d] + a$ $2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \ldots + [2a+(n-1)d] + [2a+(n-1)d]$ $= n[2a+(n-1)d]$ $S_n = \frac{n}{2}[2a+(n-1)d]$

QUESTION 3

3.1	$T_n = (8x^2)\left(\frac{x}{2}\right)^{n-1}$

3.2	$ratio = \frac{x}{2}$ $-1 < \frac{x}{2} < 1$ $-2 < x < 2$
3.3	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8x^2}{1-\frac{x}{2}}$ $S_\infty = \frac{8\left(\frac{3}{2}\right)^2}{1-\frac{1}{2}\left(\frac{3}{2}\right)}$ $S_\infty = 72$

	OR $18 + \frac{27}{2} + \frac{81}{8} + \ldots$ $S_\infty = \frac{18}{1-\frac{3}{4}}$ $S_\infty = \frac{18}{\frac{1}{4}}$ $S_\infty = 72$

PAPER I

QUESTION 2

2.1	20 ; 24 ; 28 ; 32 ; … 4 4 4 $T_n = 20 + (n-1)\,4$ $100 = 20 + 4n - 4$ $4n = 84$ $n = 21$ On the 21st day she will cycle 100 km.
	OR $T_n = 4n + 16$ $100 = 4n + 16$ $4n = 84$ $n = 21$ On the 21st day she will cycle 100 km. **OR** $100 = 20 + 80$ $= 20 + 4(21-1)$ $\therefore n = 21$
2.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{14} = \frac{14}{2}[2(20) + (14-1)4]$ $= 644$ km
2.3	No. It will not be humanly possible to just keep on increasing the distance covered indefinitely. For example: $T_{1\,000} = 4\,016$ km in one day.

QUESTION 3

3.1	45
3.2	$T_n = an^2 + bn + c$ Second difference of terms is 2. $a = 1$ $3a + b = 7$ $3 + b = 7$ $b = 4$ $a + b + c = 5$ $1 + 4 + c = 5$ $c = 0$ $T_n = n^2 + 4n$
	OR $T_n = an^2 + bn + c$ Second difference of terms is 2. $a = 1$ $T_0 = 0 = c$ $T_n = n^2 + bn + 0$ $5 = (1)^2 + (1)b$ $b = 4$ $T_n = n^2 + 4n$ **OR** If $T_n = an^2 + bn + c$ $5 = T_1 = a + b + c$ $12 = T_2 = 4a + 2b + c$ $\Rightarrow 3a + b = 7$ $21 = T_3 = 9a + 3b + c$ $\Rightarrow 5a + b = 9$ $\Rightarrow a = 1,\ b = 4,\ c = 0$

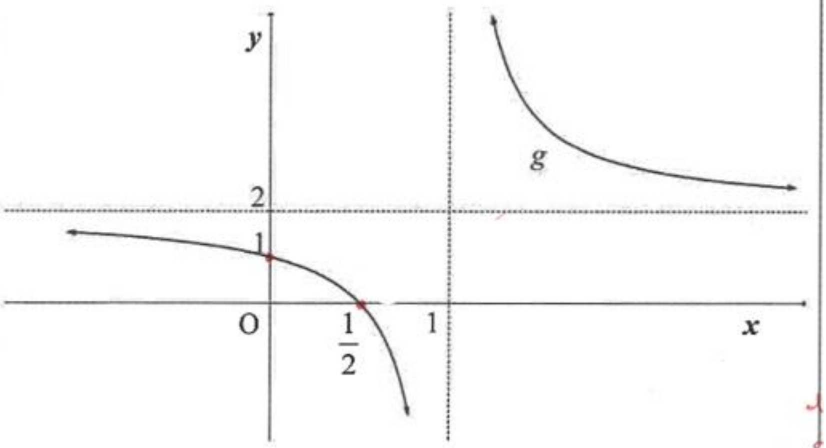
QUESTION 4

4.1	$S = a + ar + ar^2 + \ldots + ar^{n-1}$ $rS = ar + ar^2 + \ldots + ar^{n-1} + ar^n$ $S - rS = a - ar^n$ $S(1-r) = a(1-r^n)$ $S = \dfrac{a(1-r^n)}{1-r}$
4.2.1	$15\ ;\ 5\ ;\ \dfrac{5}{3}\ ;\ \ldots$ $r = \dfrac{5}{15} = \dfrac{1}{3}$ The series converges because $-1 < r < 1$
4.2.2	$S_\infty = \dfrac{15}{1-\dfrac{1}{3}}$ $= \dfrac{45}{2}$

4.3.1	$S_{24} = 2^{24+2} - 4$ $= 67108860$
4.3.2	$S_{24} = 2^{24+2} - 4 = 67108860$ $S_{23} = 2^{23+2} - 4 = 33554428$ $T_{24} = 33554432$ **OR** $T_{24} = S_{24} - S_{23}$ $= 2^{26} - 2^{25}$ $= 2 \times 2^{25} - 2^{25}$ $= 2^{25}$

FUNCTIONS AND GRAPHS

PAPER A

4.1	$x=1$ $y=2$
4.2	
4.3	$x<\frac{1}{2}$ or $x>1$ **or/of** $\left(-\infty;\frac{1}{2}\right)$ or $(1;\infty)$
4.4	$y=-(x-1)+2$ $y=-x+3$ **OR/*OF*** $y-2=-(x-1)$ $y=-x+3$ **OR/*OF*** $y=-x+c$ $2=-(1)+c$ $c=3$ $\therefore y=-x+3$

5.1	$P'(2;4)$
5.2	$f(x)=\log_a x$ $2=\log_a 4$ $a^2=4$ $a=2$

5.3	$y = 2^x$
5.4	$1 = \log_2 x$ $\therefore x = 2 \qquad T(2\,;1)$ $RT = 2$ units $P'T = 3$ units Area of $\Delta RTP' = \frac{1}{2}.RT.TP'$ $= \frac{1}{2} \times 2 \times 3 = 3 \text{ units}^2$

6.1	$y \geq -4$ or $y \in [-4;\infty)$
6.2	$x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ $\therefore E(3\,;0)$ and $D(-1\,;0)$
6.3	$P(0\,;-3)$ $\therefore m_g = 1$ $\therefore g(x) = x - 3$
6.4	$f(x) > g(x)$ $x < 0$ or $x > 3$
6.5	Distance $= -x^2 + 2x + 3 - x + 3 = -x^2 + x + 6$ $D' = -2x + 1 = 0$ or/of $x = -\frac{b}{2a} = -\frac{1}{2(-1)}$ $\therefore x = \frac{1}{2} \qquad \therefore x = \frac{1}{2}$ $D\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$ $= \frac{25}{4} = 6{,}25$

6.6	$f'(x) = m_g$ $2x - 2 = 1$ $x = \frac{3}{2}$ Point on f and k: $\left(\frac{3}{2}; \frac{-15}{4}\right)$ $k(x) = g(x) - n \quad \therefore -\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$ $\therefore n = 2\frac{1}{4} = \frac{9}{4} = 2{,}25$

	OR/OF $f(x) = k(x)$ $x^2 - 2x - 3 = x - 3 - n$ $x^2 - 3x + n = 0$ $\Delta = b^2 - 4ac$ $= (-3)^2 - 4(1)(n)$ To touch : $\Delta = 0$ $0 = 9 - 4n$ $4n = 9$ $n = \frac{9}{4} = 2{,}25$

PAPER B

4.1	$y = -4$
4.2	x – intercept: $0 = 2^x - 4$ $4 = 2^x$ $x = 2$ $\therefore B(2;0)$
4.3	$y = 2^0 - 4 = -3$ $\therefore A(0;-3)$ $y = mx + c$ $m = \frac{3}{2}$ $k(x) = \frac{3}{2}x - 3$
4.4	$k(1) = \frac{3}{2}(1) - 3 = \frac{-3}{2}$ $f(1) = 2^1 - 4 = -2$ Vertical distance $= -\frac{3}{2} - (-2) = \frac{1}{2}$ units
4.5	$g(x) = f(x) + 4$ $g(x) = 2^x$; $x \in [-2;4)$
4.6	Range of g : $y \in \left[\frac{1}{4};16\right)$ Domain of g^{-1} : $x \in \left[\frac{1}{4};16\right)$ or/*of* $\frac{1}{4} \le x < 16$

4.7	$g : y = 2^x$ $g^{-1} : x = 2^y$ $g^{-1}(x) = \log_2 x,\ x \in \left[\frac{1}{4}; 16\right)$
5.1	$(1 ; 8)$
5.2	$y = -\frac{1}{2}(0-1)^2 + 8$ $= 7\frac{1}{2}$ $C\left(0 ; \frac{15}{2}\right)$
5.3	$8 = \frac{d}{1}$ $\therefore d = 8$
5.4	$y \in R ; y \neq 0$
5.5	$-3 \leq x < 0$ or $x \geq 5$ **OR/*OF*** $x \in [-3 ; 0) \cup [5 ; \infty)$
5.6	$-2x + k = \frac{8}{x}$ $-2x^2 + kx - 8 = 0$ $\Delta = (k)^2 - 4(-2)(-8)$ $k^2 - 64 < 0$ $CV : k = 8 \ ;\ k = -8$ $\therefore -8 < k < 8$ **or/of** $k \in (-8 ; 8)$
	OR/*OF* $g^{/}(x) = h^{/}(x)$ $-\frac{8}{x^2} = -2$ $-8 = -2x^2$ $x = \pm 2$ $y = \pm 4 \quad \therefore$ B(2 ; 4) and A(– 2 ; – 4) For tangents: $h(x) = -2x + k$ or $h(x) = -2x + k$ $4 = -2(2) + k \qquad -4 = -2(-2) + k$ $k = 8 \qquad k = -8$ $\therefore -8 < k < 8$ or/*of* $k \in (-8 ; 8)$

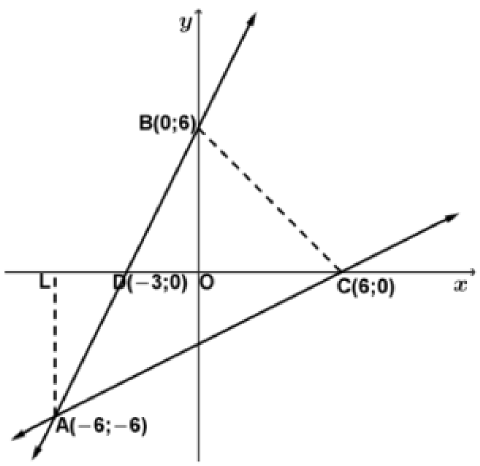
5.7	$h(x)=-2x+8$ $-2x+8=\frac{8}{x}$ $-2x^2+8x=8$ $-2x^2+8x-8=0$ $x^2-4x+4=0$ $(x-2)^2=0$ $\therefore x=2$ $f(2)=\frac{15}{2}$ $h(2)=4$ $4=\frac{15}{2}+t$ $\therefore t=-\frac{7}{2}$
	OR/*OF* $f(2)=\frac{15}{2}$ Tangent point of contact (2 ; 4) $\therefore 4=-\frac{1}{2}(2-1)^2+8+t$ $4=\frac{15}{2}+t$ $\therefore t=-\frac{7}{2}$ **OR/*OF*** $g(x)=8x^{-1}$ $g'(x)=-8x^{-2}$ $-2=-8x^{-2}$ $\frac{1}{4}=\frac{1}{x^2}$ $x=2$ $y=\frac{8}{2}=4$ $R(2;4)$ $y=-\frac{1}{2}(x-1)^2+8+t$ $4=-\frac{1}{2}(2-1)^2+8+t$ $t=-\frac{7}{2}$

PAPER C

4.1.1	$p = -1$ and $q = 2$
4.1.2	$\frac{1}{x-1} + 2 = 0$ $-2x + 2 = 1$ $x = \frac{1}{2}$ $\left(\frac{1}{2} ; 0\right)$
4.1.3	$x = \frac{1}{2} - 3$ $= \frac{-5}{2}$ Answer only: full marks
4.1.4	$y = x + t$ $2 = 1 + t$ $t = 1$
4.1.5	$-2 \leq \frac{1}{x-1}$ Answer only: full marks $\frac{1}{x-1} + 2 \geq 0$ $\therefore x \leq \frac{1}{2}$ or $x > 1$ **OR/*OF*** $x \in \left(-\infty ; \frac{1}{2}\right]$ or $(1 ; \infty)$
4.2.1	$y = -5$
4.2.2	$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$ $f(2) = 2^2 - 4(2) - 5 = -9$ $\therefore D(2 ; -9)$ **OR/*OF*** $f'(x) = 2x - 4$ $2x - 4 = 0$ $x = 2$ $f(2) = 2^2 - 4(2) - 5 = -9$ $\therefore D(2 ; -9)$

4.2.3	$q=-5$ $-9=a(2)^2-5$ $-4=4a$ $a=-1$ $\therefore g(x)=-2^x-5$
4.2.4	$y\in(-\infty;-5)$ OR $y<-5; y\in R$
4.2.5	$k<-9$

5.1	$g(x)=2x+6$ $y=6$
5.2	$y=2x+6$ $x=2y+6$ $y=\frac{1}{2}x-3$ Answer only: Full marks
5.3	$\frac{1}{2}x-3=2x+6$ $x-6=4x+12$ $3x=-18$ $x=-6$ A$(-6;-6)$ **OR/*OF*** $2x+6=x$ $x=-6$ $y=-6$
5.4	$AB=\sqrt{(6)^2+(12)^2}$ $=\sqrt{180}=6\sqrt{5}=13{,}42$



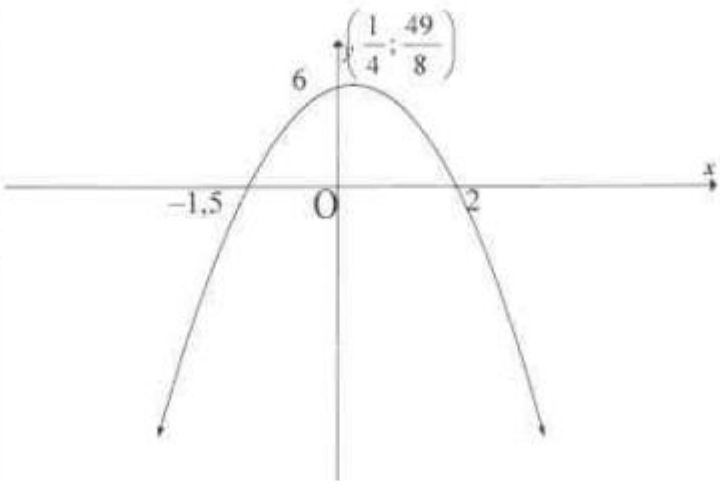
5.5	$BC=\sqrt{6^2+6^2}=\sqrt{72}=6\sqrt{2}$ $AB=AC=\sqrt{180}$ symmetry of g and g^{-1} $\perp h=(\sqrt{180})^2-\left(\frac{\sqrt{72}}{2}\right)^2$ $=\sqrt{162}=9\sqrt{2}$ area of $\Delta ABC=\frac{1}{2}BC\times h$ $=\frac{1}{2}\times\sqrt{72}\times\sqrt{162}=54$ units^2 **OR/*OF*** $\tan B\hat{D}C=2$ $\therefore B\hat{D}C=63{,}43°$ $\tan D\hat{C}A=\frac{1}{2}$ $\therefore D\hat{C}A=26{,}57°$ $\therefore D\hat{A}C=36{,}86°$ (ext angle triangle) Area of $\Delta ABC=\frac{1}{2}\left(\sqrt{180}\right)\left(\sqrt{180}\right)\sin 36{,}86°$ $=53{,}99$ units^2
	OR/OF Area of ΔABC = Area of ΔBDC + Area of ΔADC $=\frac{1}{2}DC.BO+\frac{1}{2}DC.height$ $=\frac{1}{2}(9)(6)+\frac{1}{2}(9)(6)$ $=54$ units^2

PAPER D

QUESTION 5

5.1	$x=-\frac{b}{2a}$ $=-\frac{1}{2(-2)}$ $=\frac{1}{4}$ $\therefore y=-2\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)+6$ $y=\frac{49}{8}$

5.2	$y=-2(0)^2+0+6$ $\therefore y$ intercept $(0;6)$

5.3	x intercepts $0=-2x^2+x+6$ $0=2x^2-x-6$ $0=(2x+3)(x-2)$ $\therefore x=2$ or $x=-\frac{3}{2}$ $(2;0)$ and $\left(-\frac{3}{2};0\right)$
5.4	
5.5	$k=\frac{49}{8}$
5.6	New/*Nuwe* turning point/*drpn.t* $\left(\frac{9}{4};\frac{57}{8}\right)$ Equation/*verg.* of h $y=-2\left(x-\frac{9}{4}\right)^2+\frac{57}{8}$

QUESTION/*VRAAG* 6

6.1	$x=-3$ and $y=-1$
6.2	$x\in R;\ x\neq -3$ OR $x\in(-\infty;-3)\cup(-3:\infty)$

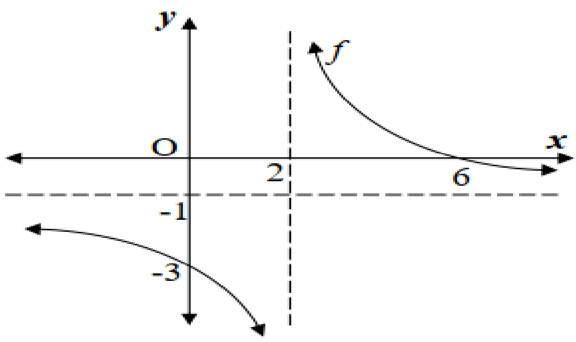
6.3.1	At B, $x = 0$ $\therefore y = \frac{1}{0+3} - 1$ $y = -\frac{2}{3}$ $\therefore$ OB $= \frac{2}{3}$ units
6.3.2	At A, $y = 0$ $0 = \frac{1}{x+3} - 1$ $1 = \frac{1}{x+3}$ $x + 3 = 1$ $x = -2$ $\therefore$ OA $= 2$ units/ eenhede
6.4	$\frac{1}{x+3} - 1 = \frac{1}{2}x$ $2 - 2(x+3) = x(x+3)$ $x^2 + 3x - 2 + 2x + 6 = 0$ $x^2 + 5x + 4 = 0$ $(x+4)(x+1) = 0$ $x = -4$ or / *of* $x = -1$ when / *wanneer* $x = -1$; $y = -\frac{1}{2}$ when / *wanneer* $x = -4$; $y = -2$ $\therefore$ C $(-1; -\frac{1}{2})$ and D $(-4; -2)$
6.5	$\frac{1}{x+3} \geq \frac{x+2}{2}$ $\frac{1}{x+3} \geq \frac{x}{2} + 1$ $\frac{1}{x+3} - 1 \geq \frac{x}{2}$ $\therefore f(x) \geq g(x)$ $\therefore x \leq -4$ or $-3 < x \leq -1$

QUESTION/*VRAAG* 7

7.1	$q = 2$ $f(x) = 2.b^{x+1} + 2$ $20 = 2.b^{1+1} + 2$ $18 = 2.b^2$ $9 = b^2$ $b = 3$ $f(x) = 2.3^{x+1} + 2$
7.2	$y = 2.3^{-1+1} + 2$ $y = 2.1 + 2$ $y = 4$
7.3	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{20-4}{1-(-1)}$ $= 8$
7.4	$h(x) = -2.3^{x+1} + 2$
7.5	$y < 2$

PAPER E

QUESTION 5

5.1	$x = 2$ *and* $y = -1$
5.2	y − intercept: $(0\ ;\ -3)$ x − intercept: $\frac{-4}{2-x} - 1 = 0$ $\frac{-4}{2-x} = \frac{1}{1}$ $-4 = 2 - x$ $x = 6$ $(6\ ;\ 0)$
5.3	

QUESTION 6

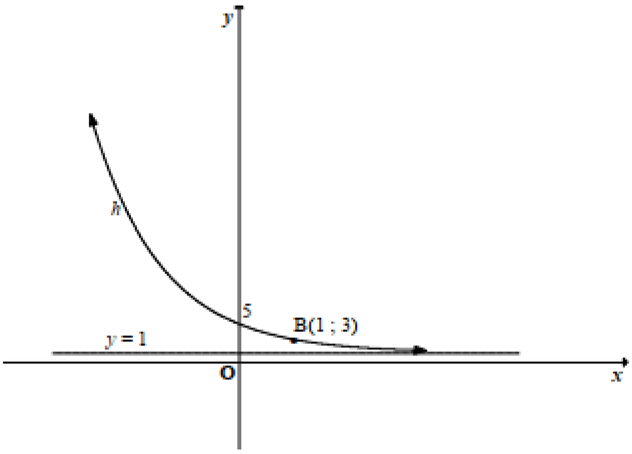
6.1	$A(0;6)$
6.2	$x=-\frac{b}{2a}=2{,}5$ $S(5\,;\,6)$
6.3	$-x^2+5x+6=0$ $x^2-5x-6=0$ $(x+1)(x-6)=0$ $x=-1\ or\ x=6$ $B(-1;0), C(6;0)$
6.4	$(-x^2+5x+6)-(x+1)=5$ $-x^2+5x+6-x-1=5$ $x^2-4x=0$ $x(x-4)=0$ $x=0\ or\ 4$ $OR = 4\text{units}$

6.5.1	$x=\frac{-1+6}{2}=\frac{5}{2}$ $y=-\left(\frac{5}{2}\right)^2+5\left(\frac{5}{2}\right)+6=\frac{49}{4}=12{,}25$ $\left(\frac{5}{2};12{,}25\right)$
6.5.2	$PQ=-x^2+4x+5$ $x=-\frac{b}{2a}=-\frac{4}{2(-1)}=2$ $\text{Max. } PQ=-(2)^2+4(2)+5=9\text{units}$

QUESTION 7

7.1	$y=5^x$
7.2	$y>0$ or $y\in(0\,;\,\infty)$
7.3	$log_5 x=-4$ $x=5^{-4}=\frac{1}{625}$ $0<x\leq\frac{1}{625}$

PAPER F

5.2.1	For y-intercept/*Vir y-afsnit* substitution $x = 0$: $y = 4.2^0 + 1$ $= 5$ H(0 ; 5)
5.2.2	For x-intercept/*Vir y-afsnit* $y = 0$ i.e./*d.i.* $4.2^{-x} + 1 = 0$ $4.2^{-x} = -1$ $2^{-x} = -\frac{1}{4}$, which is impossible, since $2^{-x} > 0$ for $x \in R$, *wat onmoontlik is omdat* $2^{-x} > 0$ *vir* $x \in R$ Therefore/*Dus*: no solution/*geen oplossing*, which means there will be no x-intercept/*wat beteken daar sal geen x-afsnit wees nie.*
5.2.3	
5.2.4	$g(x) = 4(2^{-x} + 2)$ $= 4.2^{-x} + 8$ The graph of h is translated 7 units upwards to form g/ *Die grafiek van h word 7 eenhede na bo getransleer om g te vorm.*

QUESTION 6

6.1	A(0; 2)
6.2	$-x^2+x+2=\frac{1}{2}x^2-x$ $0=\frac{3}{2}x^2-2x-2$ $0=3x^2-4x-4$ $(3x+2)(x-2)=0$ $x=-\frac{2}{3}$ or $x=2$ $y=\frac{8}{9}$ or $y=0$ $C\left(-\frac{2}{3};\frac{8}{9}\right)$ & $D(2;0)$
6.3	$x\le-\frac{2}{3}$ or $x\ge 2$
6.4	Length of PQ $=-x^2+x+2-\left(\frac{1}{2}x^2-x\right)$ $L=-\frac{3}{2}x^2+2x+2$ $x=-\frac{b}{2a}=-\frac{2}{2\left(\frac{-3}{2}\right)}=\frac{2}{3}$ OR $L'=-3x+2=0\ \therefore x=\frac{2}{3}$ Maximum value of PQ $=-\frac{3}{2}\left(\frac{2}{3}\right)^2+2\left(\frac{2}{3}\right)+2$ $=\frac{8}{3}=2\frac{2}{3}=2{,}67$ units
6.5	$f(x)=-x^2+x+2$ $f'(x)=-2x+1=3$ $x=-1$

6.6	Axis of symmetry: $x = \frac{1}{2}$ Maximum value: $y = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 2 = 2\frac{1}{4}$ $2 < k < 2\frac{1}{4}$

QUESTION/*VRAAG* 7

7.1	$CD = 2x + 3 - \left(-2x^2 + 14x + k\right)$ $= 2x + 3 + 2x^2 - 14x - k$ $= 2x^2 - 12x + 3 - k$
7.2	Minimum value occurs at/*Minimum waarde vind plaas by* $x = \frac{-b}{2a}$ $= \frac{12}{2(2)}$ $= 3$ Minimum value/*Minimum waarde* $5 = 2(3)^2 - 12(3) + 3 - k$ $5 = 18 - 36 + 3 - k$ $k = -20$

PAPER G

QUESTION 6	
6.1	$f(x) = -2x^2 - 5x + 3$ $x = \frac{-(-5)}{2(-2)}$ $x = -\frac{5}{4}$ $f(\frac{-5}{4}) = -2(\frac{-5}{4})^2 - 5(\frac{-5}{4}) + 3$ $f(\frac{-5}{4}) = \frac{49}{8}$ $\therefore TP(\frac{-5}{4}; \frac{49}{8})$

6.2	$y\in(-\infty;\frac{49}{8}]$ **OR** $y\leq\frac{49}{8}$
6.3	$\tan 135^0=-1$ $m=-1$ $f'(x)=-1$ $-4x-5=-1$ $\therefore x=-1$ $f(-1)=-2(-1)^2-5(-1)+3$ $y=6$ $\therefore P(-1\ ;\ 6)$
6.4	$k<-\frac{49}{8}$ **or** $k>-\frac{49}{8}$

QUESTION 7	
7.1	$f(x)=a^x$ $\frac{1}{4}=a^2$ $\sqrt{\frac{1}{4}}=a$ $\frac{1}{2}=a$
7.2	$y=\left(\frac{1}{2}\right)^x$ $x=\left(\frac{1}{2}\right)^y$ $y=\log_{\frac{1}{2}}x$
7.3	$y=\left(\frac{1}{2}\right)^x$ $-y=\left(\frac{1}{2}\right)^x$ $h(x)=-\left(\frac{1}{2}\right)^x$ **OR** $h(x)=-a^x$ **OR** $h(x)=-f(x)$
7.4	$x\leq 0$ **OR** $x>0$ **OR** $x<0$ **OR** $x\geq 0$

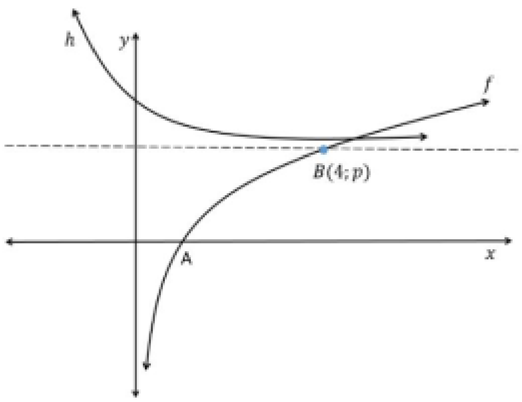
QUESTION 8		
8.1	8.1.1	$x < 2$
	8.1.2	$0 < x \leq 1$
8.2	8.2.1	$g(x) = \log_2 x$ $y = \log_2 x$ $\therefore x = 2^y$ $\therefore y = 2^x$ $\therefore g^{-1}(x) = 2^x$
	8.2.2	$\log_2(3-x) = x$ $2^x = 3 - x$ therefore point of intersection of g^{-1} and f
	8.2.3	$x = 1$

PAPER H

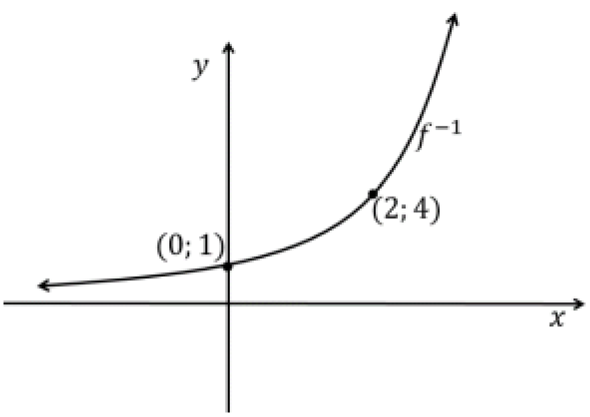
5.1	$g(x) = -3x + 20$ $g(3) = -3(3) + 20 = 11$ $\therefore k = 11$
5.2	$y \geq -11$ OR $x \in [-11; \infty)$
5.3	$y = a(x-3)^2 + 11$ $subst. (6; 2)$ $2 = a(6-3)^2 + 11$ $-9 = 9a$ $-1 = a$ $y = -1(x-3)^2 + 11$ $y = -(x^2 - 6x + 9) + 11$ $y = -x^2 + 6x + 2$ $\therefore a = -1 \,; b = 6 \; and \; /en \;\; c = 2$

5.4	$3 < x < 6$ OR $x \in (3; 6)$
5.5	• Real *Reëel* • Rational *Rasionaal* • Equal *Gelyk*
5.6	$x > 3$ OR $x \in (3; \infty)$

QUESTION 6



6.1	$A(1; 0)$
6.2	$x \in R$; $x > 0$ or $x \in (0; \infty)$
6.3	$x = \log_2 y$ $y = 2^x$

6.4	

6.5	$p = \log_2 4 = 2$ $\therefore y = 2$
6.6	Reflection in the y −axis and translate 2 units down.

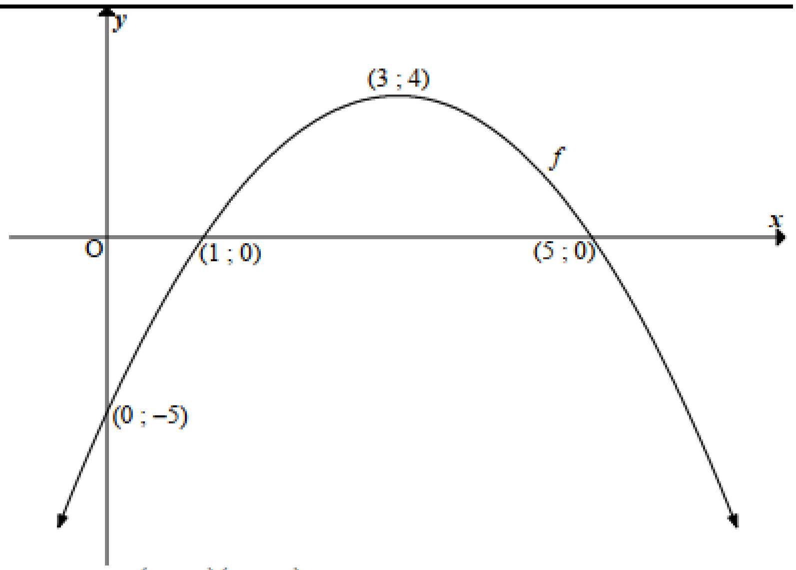
PAPER I

QUESTION 5

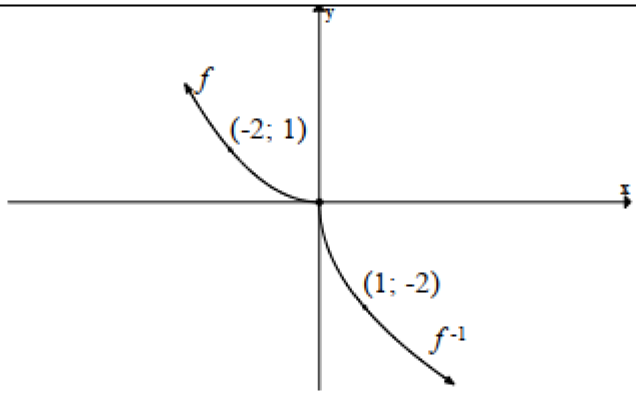
5.1.1	$C(0\,;-3)$
5.1.2	$f(x) = x^2 - 2x - 3$ $(x-3)(x+1) = 0$ $x = -1$ or $x = 3$ $AB = 3-(-1)$ $AB = 4$ units
5.1.3	$x = \dfrac{2}{2(1)}$ or $2x - 2 = 0$ or $x = \dfrac{-1+3}{2}$ $= 1$ $y = (1)^2 - 2(1) - 3$ $= -4$ $D(1\,;-4)$

5.1.4	C(0 ; –3) D(1 ; – 4) Average gradient / *Gemiddelde gradiënt* $=\frac{-4+3}{1-0}$ or $\frac{-3+4}{0-1}$ $=-1$
5.1.5	OC = OB = 3 $\hat{OCB}=45°$ isosceles right angled triangle *Gelykbenige reghoekige driehc* **OR / OF** $\tan\beta = m_g$ $\tan\beta = 1$ $\beta = 45°$ $\hat{OBC}=45°$ $\hat{OCB}=45°$
5.1.6	$-4<k<-3$ **OR** (–4 ; –3)

5.1.7	$f'(x)\ .f''(x)>0$ $(2x-2).2>0$ $2x-2>0$ $x>1$

5.2	 $f(x)=a(x-1)(x-5)$ $4=a(3-1)(3-5)$ $4=-4a$ $a=-1$ $f(x)=-x^2+6x-5$

QUESTION/*VRAAG* 6

6.1	$f:\quad y=\frac{1}{4}x^2$ $f^{-1}:\quad x=\frac{1}{4}y^2$ $y^2=4x$ $y=\pm\sqrt{4x}$ $f^{-1}(x)=-\sqrt{4x}$ **OR**/OF $f^{-1}(x)=-2\sqrt{x}$
6.2	

6.3	Yes. No value of x in the domain of f^{-1} maps onto more than one y-value. *Ja. Geen waarde van x in die definisieversameling van f^{-1} assosieer met meer as een y-waarde nie.* **OR/*OF*** Yes. One to one function./*Ja. Een-tot-een-funksie.* **OR/*OF*** Yes. Vertical line test holds./*Ja. Die vertikale lyntoets werk.*

FINANCE, GROWTH AND DECAY

PAPER A

7.1	$A = P(1-i)^n$ $8\,337{,}75 = 13\,000(1-i)^6$ $i = 0{,}0714\ldots$ $r = 7{,}14\%$
7.2	$F = \dfrac{x\left[(1+i)^n - 1\right]}{i}$ $80\ 000 = \dfrac{x\left[\left(1+\frac{8,6}{1200}\right)^{36} - 1\right]}{\frac{8,6}{1200}}$ $x = \text{R}1\ 955{,}78$ Thandi's total = 1 955,78 × 36 = R 70 408,08 Eric's total = 1 402,31 × 48 = R 67 310,88 Difference = 70 408,08 – 67 310,88 = R3 097,20
7.3	$225\ 000\left(1+\dfrac{0,09}{12}\right)^3 = \dfrac{5\ 500\left[1-\left(1+\frac{0,09}{12}\right)^{-n}\right]}{\frac{0,09}{12}}$ $0{,}3137734959\ldots = 1-\left(1+\dfrac{0,09}{12}\right)^{-n}$ $\left(1+\dfrac{0,09}{12}\right)^{-n} = 0{,}6862265041\ldots$ $-n = \log_{\left(1+\frac{0,09}{12}\right)} 0{,}6862265041\ldots$ $n = 50{,}394375\ldots$ $n = 51$

PAPER B

6.1.1	$A = P(1+i)^n$ $19\ 319{,}48 = 18\ 500\left(1+\frac{r}{1200}\right)^6$ $\left(1+\frac{r}{1200}\right) = \sqrt[6]{1{,}04429...}$ $\frac{r}{1200} = 0{,}00725...$ $r = 8{,}7\%$
6.1.2	$1+\frac{i}{100} = \left(1+\frac{8{,}7}{1200}\right)^{12}$ $r = 9{,}06\%$
6.2.1	$A = P(1-in)$ $0 = 10\ 000(1-0{,}2n)$ $n = 5$
6.2.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $20\ 000 = \frac{x\left[\left(1+\frac{8{,}7}{1200}\right)^{60} - 1\right]}{\frac{8{,}7}{1200}}$ $x = R267{,}26$
6.3	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $1\ 600\ 000 = \frac{20\ 000\left[1-\left(1+\frac{0{,}112}{12}\right)^{-n}\right]}{\frac{0{,}112}{12}}$ $\frac{56}{75} = 1-\left(1+\frac{0{,}112}{12}\right)^{-n}$ $\left(1+\frac{0{,}112}{12}\right)^{-n} = \frac{19}{75}$ $-n = \log_{\left(1+\frac{0{,}112}{12}\right)}\left(\frac{19}{75}\right)$ $-n = -147{,}80$ Tino will make 147 withdrawals of R20 000

PAPER C

6.1	$A = P(1+i)^n$ $13\ 459 = 12\ 000\left(1+\frac{m}{400}\right)^8$ $\left(1+\frac{m}{400}\right)^8 = 1{,}121...$ $1+\frac{m}{400} = \sqrt[8]{1{,}121..}$ $\frac{m}{400} = 0{,}0144...$ $\therefore m = 5{,}78\%$
6.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $F = \frac{1\ 000\left[\left(1+\frac{0{,}075}{12}\right)^{12} - 1\right]}{\frac{0{,}075}{12}}$ $= R12\ 421{,}22$ He won't be able to buy the computer because R13 000 – R12 421,22 = R578,78 **OR/OF** He won't be able to buy the computer because R12 421,22 < R13 000
6.3.1	Loan amount = 85% × R250 000 = R212 500 **OR/OF** Loan amount = R250 000 – (15% × R250 000) = R212 500
6.3.2	$A = 212\ 500\left(1+\frac{0{,}13}{12}\right)^5$ $A = 224\ 262{,}53$ $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $224\ 262{,}53 = \frac{x\left[1-\left(1+\frac{0{,}13}{12}\right)^{-67}\right]}{\frac{0{,}13}{12}}$ $\therefore x = R4\ 724{,}96$

PAPER D

7.1	$A = P(1+i)^n$ $23000 = 1570(1.12)^n$ $(1.12)^n = 14,64968153..$ $n\log(1,12) = \log 14,64968153..$ $n = 23,69$ years or $n = 24$ years or $n = 23$ years 8 months or $n = 23,7$ years
7.2.1	$A = P(1+i)^n$ $= 800000(1.08)^5$ $= R1175462,46$ $\therefore R1175462,46 - R200\ 000$ $= R975462,46$ Some calculators give R 975 462,50
7.2.2	$F = \dfrac{x[(1+i)^n - 1]}{i}$ $975462,46 = x\dfrac{[1,01]^{60} - 1}{0,01}$ $\dfrac{975462,46 \times 0.01}{[1,01]^{60} - 1} = x$ $x = R\,11944,00$

7.2.3	$Service = [5000(1{,}01)^{48} + 5000(1{,}01)^{36} + 5000(1{,}01)^{24} + 5000(1{,}01)^{12} + 5000]$ $= 32197{,}77$ $975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01} - Service$ $975462{,}46 = 81{,}66966986x - 32197{,}77$ $x = R\,12338{,}24$ **OR** $Service = \frac{5000[1{,}01^{60} - 1]}{1{,}01^{12} - 1}$ $= 32197{,}77$ $975462{,}46 = x\frac{[1{,}01]^{60} - 1}{0{,}01} - Service$ $975462{,}46 = 81{,}66966986x - 32197{,}77$ $x = R\,12338{,}24$

PAPER E

QUESTION 7

7.1	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log\frac{1}{2} = n\log 0{,}93$ $n = \frac{\log\frac{1}{2}}{\log 0{,}93}$ $= 9{,}55$ years	**OR**	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log_{0{,}93}\frac{1}{2} = n$ $n = 9{,}55$ years

7.2	**Radesh:** $A = P(1+in)$ $= 6\,000(1+0{,}085\times 5)$ $= 8\,550$ OR $A = 6\,000 + 8{,}5\% \text{ of } 6000\times$ $= 6000 + 510\times 5$ $= 6000 + 2550$ $= 8\,550$ $\text{Bonus} = 0{,}05\times 6\,000$ $= 300$ $\text{Received} = 8\,550 + 300$ $= \text{R}8\,850$ **Thandi:** $A = P(1+i)^n$ $= 6\,000\left(1+\frac{0{,}08}{4}\right)^{20}$ $= \text{R}8\,915{,}68$ Thandi's investment is bigger.
7.3	$F_v = \text{initial deposit with interest + annuity}$ $= 1\,000\left(1+\frac{0{,}15}{12}\right)^{18} + 700\left(\frac{\left(1+\frac{0{,}15}{12}\right)^{18}-1}{\frac{0{,}15}{12}}\right)$ $= 1\,250{,}58 + 14\,032{,}33$ $= \text{R}15\,282{,}91$

PAPER F

QUESTION 8

8.1	$\text{Depreciation value} = 7\ 200(1-0{,}25)^3$ $= R3\ 037{,}50$

8.2.1	$300\ 000 = \frac{5\ 000\left[1-(1{,}015)^{-n}\right]}{0{,}015}$ $4\ 500 = 5\ 000 - 5\ 000(1{,}015)^{-n}$ $5\ 000(1{,}015)^{-n} = 500$ $(1{,}015)^{-n} = 0{,}1 \quad \text{or} \quad (1{,}015)^{n} = 10$ $-n = \frac{\log 0{,}1}{\log 1{,}015}$ $n = 154{,}65$ Number of payments = 155
8.2.2	Balance outstanding $= 300\ 000\left(1+\frac{0.18}{12}\right)^{154} - \frac{5\ 000\left[\left(1+\frac{0.18}{12}\right)^{154}-1\right]}{\frac{0.18}{12}}$ $= R3\ 230{,}50$

8.2.3	Amount paid in last month $= 3\ 230{,}50\left(1+\frac{0.18}{12}\right)$ $= R3\ 278{,}96$
8.2.4	Total repaid = (154 × 5 000) + 3 278,96 = R773 278,96

PAPER G

QUESTION 8

8.1	$A = P(1-i)^n$ $250000 = P(1-13{,}5\%)^5$ $P = \dfrac{250000}{(1-13{,}5\%)^5}$ $= R516249$
8.2.1	$P = \dfrac{x[1-(1+i)^{-n}]}{i}$ $950000 = \dfrac{x\left[1-\left(1+\frac{14{,}25\%}{12}\right)^{-240}\right]}{\frac{14{,}25\%}{12}}$ $x = R11986{,}33$
8.2.2	$P = \dfrac{x[1-(1+i)^{-n}]}{i}$ $= \dfrac{11986{,}33\left[1-\left(1+\frac{14{,}25\%}{12}\right)^{-140}\right]}{\frac{14{,}25\%}{12}}$ $= R816048{,}67$ **OR** $A = P(1+i)^n$ $A = 950\,000\left(1+\dfrac{14{,}25\%}{12}\right)^{100}$ =R3093215,766 $F = \dfrac{x[(1+i)^n-1]}{i}$ $F = \dfrac{11986{,}33\left[\left(1+\frac{14{,}25\%}{12}\right)^{100}-1\right]}{\frac{14{,}25\%}{12}}$ =R2277167,107 Balance on Loan = R3093215,766 –R2277167,107 = R816048,67

8.2.3	$A=P(1+i)^n$ $=816\ 048{,}67\left(1+\frac{14{,}25\%}{12}\right)^4$ $=R855\ 506{,}92$ $855\ 506{,}92=\frac{x[1-(1+i)^{-n}]}{i}$ $=\frac{x\left[1-\left(1+\frac{14{,}25\%}{12}\right)^{-136}\right]}{\frac{14{,}25\%}{12}}$ $x=R12711{,}51$

PAPER H

QUESTION 8

8.1	$A=P(1+i)^n$ $3P=P\left(1+\frac{i}{12}\right)^{72}$ $i=12(\sqrt[72]{3}-1)$ $i=0{,}1845$ Annual interest rate is 18,45 % p.a.
8.2.1	$P=\frac{x[1-(1+i)^{-n}]}{i}$ $192\ 000=\frac{x\left[1-\left(1+\frac{0.12}{12}\right)^{-60}\right]}{\frac{0.12}{12}}$ $x=R4270{,}93$
8.2.2	$P=\frac{x[1-(1+i)^{-n}]}{i}$ $=\frac{4270{,}93\left[1-\left(1+\frac{0.12}{12}\right)^{-15}\right]}{\frac{0.12}{12}}$ $=R59216{,}72421$

	OR $A = P(1+i)^n$ $A = 192\ 000\left(1+\frac{0.12}{12}\right)^{45}$ $=R300\ 443{,}6635$ $F = \frac{x[(1+i)^n - 1]}{i}$ $F = \frac{4270{,}934\left[\left(1+\frac{0.12}{12}\right)^{45} - 1\right]}{\frac{0.12}{12}}$ $=R241\ 226{,}9424$ Balance on Loan $= R300\ 443{,}6635 - R241\ 226{,}9424$ $= R59216{,}7211$

PAPER I

QUESTION 4	
4.1	$4500 = 3000\left(1+\frac{0,08}{12}\right)^n$ $\frac{3}{2} = \left(1+\frac{0,08}{12}\right)^n$ $\log_{\left(1+\frac{0,08}{12}\right)}\frac{3}{2} = n$ $n = 61,02$ months / (accept 62) $n = 5,09$ years / (accept 5,17) **NOTE: (5,08 is NOT accepted.)**
4.2.1	$P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $40\ 000 = \frac{x[1-(1+\frac{0,24}{12})^{-240}]}{\frac{0,24}{12}}$ $x = R806,96$

4.2.2	$P_o = \dfrac{x[1-(1+i)^{-n}]}{i}$ $P_o = \dfrac{806,96\left[1-\left(1+\dfrac{0,24}{12}\right)^{-180}\right]}{\dfrac{0,24}{12}}$ $= R39205,67$ **OR** $P_o = P(1+i)^n - \dfrac{x[(1+i)^n - 1]}{i}$ $P_o = 40000(1+\dfrac{0,24}{12})^{60} - \dfrac{806,96\left[\left(1+\dfrac{0,24}{12}\right)^{60} - 1\right]}{\dfrac{0,24}{12}}$
	$= R39\,206,20$

4.2.3	$P_v = \dfrac{x[1-(1+i)^{-n}]}{i}$ $39205,67 = \dfrac{x[1-(1+\dfrac{0,18}{12})^{-180}]}{\dfrac{0,18}{12}}$ $x = \text{R}631,38$ **NOTE: If the candidate uses R39 206,20 then the answer is R631,39.**

PAPER J

QUESTION 7

7.1	$1+i_{eff} = \left(1+\frac{0,11}{2}\right)^2$ $i_{eff} = \left(1+\frac{0,11}{2}\right)^2 - 1$ $i_{eff} = 11,30\%$ $\therefore$ Mary has secured the better rate.
7.2.1	$Fv = \dfrac{10\ 000\left[\left(1+\frac{0,0772}{12}\right)^{114} - 1\right]}{\frac{0,0772}{12}}$ $= R1\ 674\ 501,44$
7.2.2	$R1\ 674\ 501,44 = \dfrac{30000\left[1-\left(1+\frac{0,1}{12}\right)^{-n}\right]}{\frac{0,1}{12}}$ $0,46513... = \left[1-\left(1+\frac{0,1}{12}\right)^{-n}\right]$ $0,53486... = \left(1+\frac{0,1}{12}\right)^{-n}$ $\log_{\left(1+\frac{0,1}{12}\right)} 0,53486... = -n$ $n = 75,4$ She will be able to receive the money in 75 full months.
7.2.3	$Pv = \dfrac{30\ 000\left[1-\left(1+\frac{0,1}{12}\right)^{-55}\right]}{\frac{0,1}{12}}$ $Pv =$ R 1 319 260,60 $\therefore$ No

DIFFERENTIAL CALCULUS

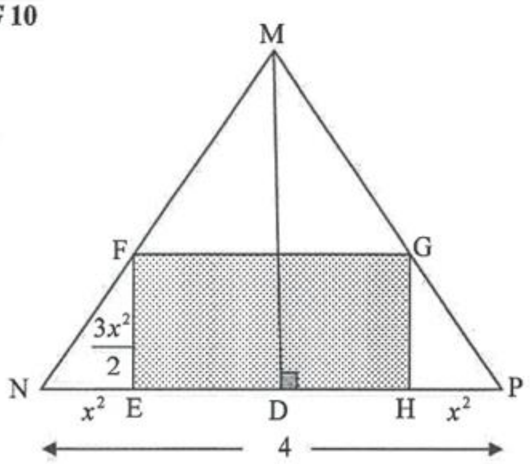
PAPER A

8.1	$f(x)=\frac{1}{x}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h}$ $f'(x)=\lim_{h\to 0}\frac{x-(x+h)}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$ $f'(x)=-\frac{1}{x^2}$ **OR/*OF***
	OR/*OF* $f(x)=\frac{1}{x}$ $f(x+h)=\frac{1}{x+h}$ $f(x+h)-f(x)=-\frac{h}{x(x+h)}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$ $f'(x)=-\frac{1}{x^2}$

8.2.1	$\frac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ $=\frac{d}{dx}\left(2x^3+\sqrt{2}.x^2\right)$ $=6x^2+2\sqrt{2}x$
8.2.2	$g(x)=\frac{3x^4-4x^2+6}{x^2}$ $g(x)=3x^2-4+6x^{-2}$ $g'(x)=6x-12x^{-3}$
8.3	$f(x)=3x^2+bx+c$ $f'(x)=6x+b$ $f'(1)=6+b=9$ $\therefore b=3$ $f(1)=3+3+c=0$ $c=-6$ $\therefore f(x)=3x^2+3x-6$

9.1	$f(x)=ax^3+bx^2+cx-5$ $-5=a(0+1)^2(0-5)$ $-5=-5a$ $a=1$ $f(x)=(x+1)(x+1)(x-5)$ $f(x)=(x^2+2x+1)(x-5)$ $f(x)=x^3-3x^2-9x-5$ $\therefore\ b=-3$ and $c=-9$
9.2	$f(x)=x^3-3x^2-9x-5$ $f'(x)=3x^2-6x-9$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x=3$ or $x=-1$ Minimum value at $x=3$
9.3	$f''(x).f(x)>0$ Point of inflection: $x=1$ $x<1$; $x\neq-1$ or $x>5$
9.4	$-32<-t<-5$ $5<t<32$ **OR/*OF*** Shift up less than 32 units $\therefore 5<t<32$

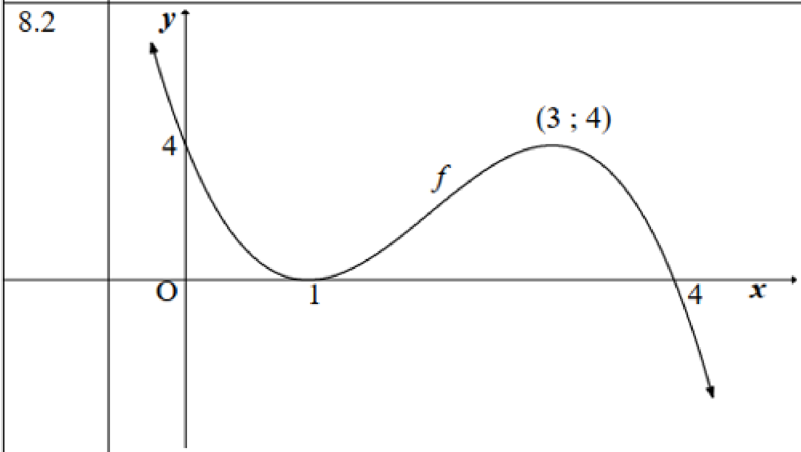
QUESTION 10/*VRAAG* 10



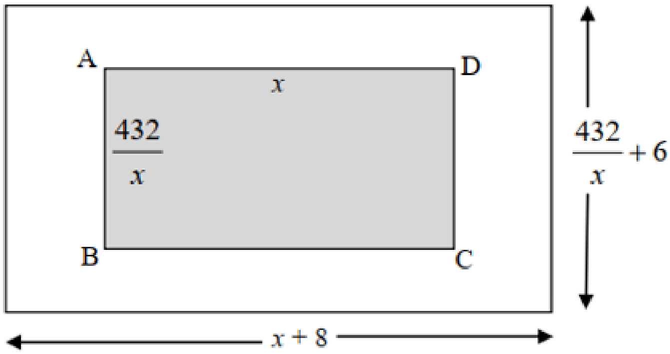
10.1	$\frac{\text{NE}}{\text{EF}} = \frac{2}{3} = \frac{x^2}{b}$ $3x^2 = 2b$ $\therefore b = \frac{3x^2}{2}$ $\text{EH} = 4 - 2x^2$ $\text{Area EFGH} = (4-2x^2)\left(\frac{3x^2}{2}\right)$ $\text{A}(x) = 6x^2 - 3x^4$ **OR/OF** In ΔDMP: $\tan P = \frac{3}{2}$ In ΔHGP: $\tan P = \frac{GH}{x^2}$ $\frac{GH}{x^2} = \frac{3}{2}$ $\therefore b = \frac{3x^2}{2}$ $\text{EH} = 4 - 2x^2$ $\text{Area EFGH} = (4-2x^2)\left(\frac{3x^2}{2}\right)$ $\text{A}(x) = 6x^2 - 3x^4$
10.2	$\text{A}(x) = 6x^2 - 3x^4$ $\text{A}'(x) = 12x - 12x^3 = 0$ $12x(1-x^2) = 0$ $\therefore x \neq 0$ or $x = -1$ or $x = 1$ $\therefore$ max area: $\text{A}(1) = 6(1)^2 - 3(1)^4 = 3$

PAPER B

7.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-4(x+h)^2-(-4x^2)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-4x^2-8xh-4h^2+4x^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{-8xh-4h^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(-8x-4h)}{h}$ $f'(x)=\lim_{h\to 0}(-8x-4h)$ $f'(x)=-8x$
	OR/*OF* $f(x+h)=-4(x+h)^2=-4x^2-8xh-4h^2$ $f(x+h)-f(x)=-4x^2-8xh-4h^2-(-4x^2)$ $=-8xh-4h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-8xh-4h^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(-8x-4h)}{h}$ $f'(x)=\lim_{h\to 0}(-8x-4h)$ $f'(x)=-8x$
7.2.1	$f(x)=2x^3-3x$ $f'(x)=6x^2-3$
7.2.2	$D_x\left[7.\sqrt[3]{x^2}+2x^{-5}\right]$ $D_x\left[7.x^{\frac{2}{3}}+2x^{-5}\right]$ $=\frac{14}{3}x^{-\frac{1}{3}}-10x^{-6}$
7.3	$-6x^2+8>0$ $x^2<\frac{8}{6}$ CV's: $x=-\frac{2}{\sqrt{3}}$ or $x=\frac{2}{\sqrt{3}}$ Positive for : $-\frac{2}{\sqrt{3}}<x<\frac{2}{\sqrt{3}}$

8.1	$f'(x) = -3x^2 + 12x - 9$ $-3x^2 + 12x - 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3$ or $x = 1$ $f(3) = -(3)^3 + 6(3)^2 - 9(3) + 4 = 4$ $f(1) = -(1)^3 + 6(1)^2 - 9(1) + 4 = 0$ $\therefore$ turning points are: (3 ; 4) and (1 ; 0)
8.2	
8.3	$0 < k < 4$ or/*of* $k \in (0 ; 4)$
8.4	$f''(x) = -6x + 12 = 0$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$ **OR/*OF*** Point of inflection: $x = \frac{3+1}{2}$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$
8.5	$\tan\theta = 3$ $\therefore \theta = 71{,}57°$

QUESTION 9/*VRAAG* 9



9.1	$432 = xb$ $\therefore b = \frac{432}{x}$ $A(x) = (x+8)\left(\frac{432}{x}+6\right)$ $A(x) = 432 + 6x + \frac{3456}{x} + 480$ $A(x) = \frac{3456}{x} + 6x + 480$
9.2	$A(x) = 3456x^{-1} + 6x + 480$ $A'(x) = -\frac{3456}{x^2} + 6$ $-\frac{3456}{x^2} + 6 = 0$ $3456 = 6x^2$ $\therefore x = \sqrt{576} = 24\text{ cm}$

PAPER C

9.1	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{(x+h)^2 - \frac{1}{2}(x+h) - \left(x^2 - \frac{1}{2}x\right)}{h}$ $= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - \frac{1}{2}x - \frac{1}{2}h - x^2 + \frac{1}{2}x}{h}$ $= \lim_{h \to 0} \frac{h\left(2x + h - \frac{1}{2}\right)}{h}$ $= 2x - \frac{1}{2}$
9.2.1	$\frac{d}{dx}[3x^4 + \sqrt[5]{x} + a^2]$ $\frac{d}{dx}\left[3x^4 + x^{\frac{1}{5}} + a^2\right]$ $= 12x^3 + \frac{1}{5}x^{-\frac{4}{5}}$
9.2.2	$xy = x + x^2 - 1$ $y = 1 + x - x^{-1}$ $\frac{dy}{dx} = 1 + x^{-2}$

QUESTION 10

10.1.1	$x^3+5x^2-8x-12=0$ $(x+1)$ is a factor $f(-1)=0$ $(x+1)(x^2-4x-12)=0$ $(x+6)(x+1)(x-2)=0$ $x=-6 \quad or \quad x=-1 \quad or \quad x=2$
10.1.2	$f(x)=x^3+5x^2-8x-12$ $f^{/}(x)=3x^2+10x-8=0$ $(3x-2)(x+4)=0$ $x=\frac{2}{3}$ or $x=-4$ $f\left(\frac{2}{3}\right)=\left(\frac{2}{3}\right)^3+5\left(\frac{2}{3}\right)^2-8\left(\frac{2}{3}\right)-12=-\frac{400}{27}$ $=-14{,}81$ $B\left(\frac{2}{3};-14{,}81\right)$
10.1.3	$f''(x)=6x+10=0$ $x=-\frac{5}{3}$ **OR** $x=\frac{\frac{2}{3}+(-4)}{2}=-\frac{5}{3}$ **OR** $x=-\frac{b}{3a}=-\frac{5}{3}$

10.2.1	$f'(0) = -8$ $y = -8x - 12$
10.2.2	$f'(x).g'(x) > 0$ Since $g'(x) < 0$ for all $x \in R$ $(3x^2 + 10x - 8) < 0$ $(3x - 2)(x + 4) < 0$ $-4 < x < \frac{2}{3}$ **OR** $-4 < x < \frac{2}{3}$

PAPER D

9.1	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - b(x+h) - (x^2 - bx)}{h}$ $f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - bx - bh - x^2 + bx}{h}$ $f'(x) = \lim_{h \to 0} \frac{2xh + h^2 - bh}{h}$ $f'(x) = \lim_{h \to 0} \frac{h(2x + h - b)}{h}$ $f'(x) = 2x - b$

9.2.1	$\frac{d}{dx}\left[\frac{x^4}{4} - 3\sqrt[3]{x} + 7\right]$ $\frac{d}{dx}\left[\frac{x^4}{4} - 3x^{\frac{1}{3}} + 7\right]$ $= x^3 - x^{-\frac{2}{3}}$
9.2.2	$y = (x^{\frac{1}{3}} - 2x^{\frac{2}{3}})^2$ $y = x^{\frac{2}{3}} - 4x + 4x^{\frac{4}{3}}$ $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} - 4 + \frac{16}{3}x^{\frac{1}{3}}$

QUESTION 10

10.1		
	10.1.1	$x = -5$ or $x = 1$
	10.1.2	$x = -4$ or $x = 0$

10.2

$y = a(x+5)(x-1)$
$-15 = a(0+5)(0-1)$
$-15 = -5a$
$3 = a$
$y = 3(x+5)(x-1) = 3x^2 + 12x - 15$

OR

$y = a(x+p)^2 + q$
$y = a(x+2)^2 + q$
$(0\,;-15)$: $-15 = a(0+2)^2 + q$
$-15 = 4a + q$ .... (1)
$(1\,;0)$: $0 = a(1+2)^2 + q$
$0 = 9a + q$ .... (2)
(2) – (1) : $15 = 5a \quad \therefore \quad a = 3$
$0 = 27 + q \quad \therefore \quad q = -27$
$y = 3(x+2)^2 - 27$
$y = 3(x^2 + 4x + 4) - 27$
$y = 3x^2 + 12x - 15$

10.3

$f(x) = ax^3 + bx^2 + cx + d$
$f^/(x) = 3ax^2 + 2bx + c$
$f^/(x) = 3x^2 + 12x - 15$
Equating coefficients of equal polynomials
$3a = 3$ and $2b = 12$ and $c = -15$
$a = 1$ and $b = 6$ and $c = -15$
$f(x) = x^3 + 6x^2 - 15x + d$

$f(-3) = (-3)^3 + 6(-3)^2 - 15(-3) + d$
$f(-3) = -27 + 54 + 45 + d$
$0 = 72 + d$
$-72 = d$

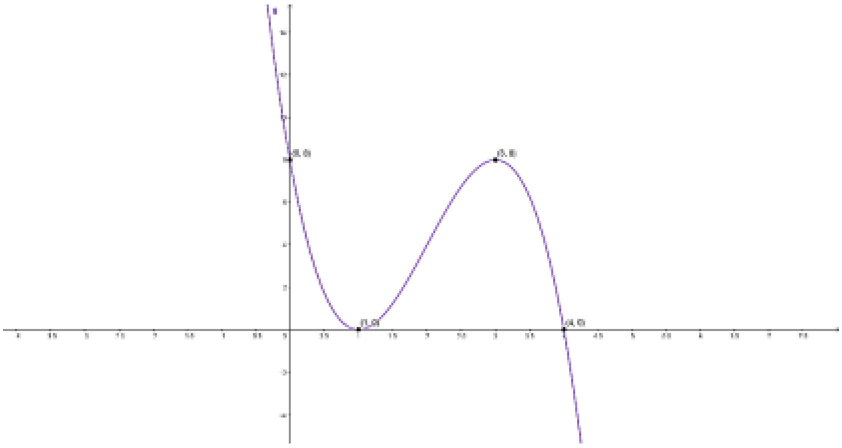
10.4	$x = -5$: $y = (-5)^3 + 6(-5)^2 - 15(-5) - 72 = 28$ $(-5\,;28)$ maximum point $x = 1$: $y = (1)^3 + 12(1)^2 - 15(1) - 72 = -74$ $(1\,;-74)$ minimum point
10.5	$3x^2 + 12x - 15 = t$ $3x^2 + 12x - 15 - t = 0$ $\Delta = b^2 - 4ac = 0$ $\Delta = (12)^2 - 4(3)(-15 - t) = 0$ $144 + 180 + 12t = 0$ $12t = -324$ $t = -27$

PAPER E

9.1	Volume of Sphere $= \frac{4}{3}\pi(8)^3$ or $= \frac{2048\pi}{3}$ or $= 2144{,}66$
9.2	$r^2 + x^2 = 8^2$ (Pythagoras) $r^2 = 64 - x^2$
9.3	$V_{cone} = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(64 - x^2)(8 + x)$ $= \frac{\pi}{3}(512 + 64x - 8x^2 - x^3)$ $\frac{dV}{dx} = \frac{64\pi}{3} - \frac{16\pi}{3}x - \frac{3\pi}{3}x^2$ $0 = 64 - 16x - 3x^2$ $0 = (8 - 3x)(x + 8)$ $x = \frac{8}{3}$ $x \neq -8$ $\frac{V_{cone}}{V_{sphere}} = \frac{\frac{1}{3}\pi\left(\frac{512}{9}\right)\left(\frac{32}{3}\right)}{\frac{2048\pi}{3}}$ $= \frac{8}{27} = 0{,}3$

QUESTION 10

10.1



10.2

$$f(x) = a(x-1)^2(x-4)$$
$$8 = a(3-1)^2(3-4)$$
$$8 = a(-4)$$
$$a = -2$$

$$f(x) = -2(x-1)^2(x-4)$$
$$f(x) = -2(x^2-2x+1)(x-4)$$
$$f(x) = -2x^3+12x^2-18x+8$$

10.3

$$f(x) = -2x^3+12x^2-18x+8$$
$$f'(x) = -6x^2+24x-18$$
$$f''(x) = -12x+24$$
$$f''(x) < 0$$
$$-12x+24 < 0$$
$$-x < -2$$
$$x > 2$$

QUESTION 12

12.1	$h'(x) = -3x^2 + 2ax + b$ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ $0 = -3 - 2a + b$ $2a - b = -3 \quad \ldots \text{(i)}$ $h'(2) = -3(2)^2 + 2a(2) + b$ $0 = -12 + 4a + b$ $4a + b = 12 \quad \ldots \text{(ii)}$ (ii) + (i): $\quad 6a = 9$ $a = \frac{3}{2}$ $\therefore 2\left(\frac{3}{2}\right) - b = -3$ $b = 6$

12.2	Average Gradient $= \frac{10 - (-3{,}5)}{2 - (-1)}$ $= \frac{13{,}5}{3}$ $= \frac{9}{2}$

12.3	$h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2\,;\,2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$
12.4	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$ **OR** $x = \frac{-1+2}{2}$ $x = \frac{1}{2}$
12.5	$p > 3{,}5$ or $p < -10$

PAPER F

QUESTION 8

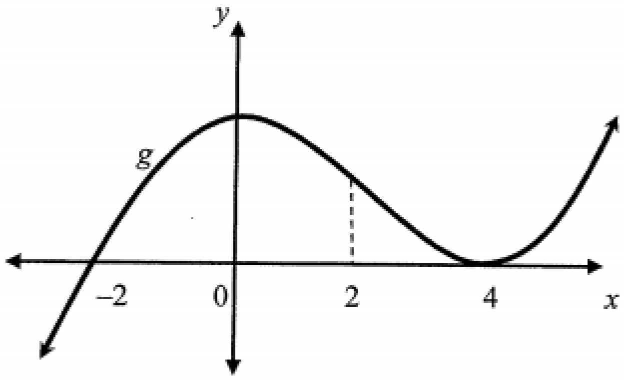
8.1	$f(x) = 2x^2 - 5x + 3$ $f(x+h) = 2(x+h)^2 - 5(x+h) + 3$ $= 2(x^2 + 2xh + h^2) - 5x - 5h + 3$ $= 2x^2 + 4xh + 2h^2 - 5x - 5h + 3$ $f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 - 5x - 5h + 3$ $- (2x^2 - 5x + 3)$ $= 4xh + 2h^2 - 5h$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{4xh + 2h^2 - 5h}{h}$ $= \lim_{h \to 0} \frac{h(4x + 2h - 5)}{h}$ $= \lim_{h \to 0} (4x + 2h - 5)$ $= 4x - 5$

8.2	$y=\frac{2x^2}{3\sqrt{x}}-\frac{2x^3+1}{x^3}$ $=\frac{2x^2}{3x^{\frac{1}{2}}}-2-\frac{1}{x^3}$ $=\frac{2}{3}x^{\frac{3}{2}}-2-x^{-3}$ $\frac{dy}{dx}=x^{\frac{1}{2}}+3x^{-4}$

QUESTION 9

9.1.1	$f(x)=-2x^3+5x^2+4x-3$ $0=(x-3)(-2x^2-x+1)$ $x-3=0$ or $-2x^2-x+1=0$ $x=3$ $\quad 2x^2+x-1=0$ $(3;0)$ $\quad (2x-1)(x+1)=0$ $2x=1$ or $x=-1$ $x=\frac{1}{2}$ $\quad (-1;0)$ $\left(\frac{1}{2};0\right)$

9.1.2	$f'(x)=-6x^2+10x+4$ $0=-6x^2+10x+4$ $3x^2-5x-2=0$ $(3x+1)(x-2)=0$ or $x=\frac{-10\pm\sqrt{10^2-4(-6)(4)}}{2(-6)}$ $3x=-1$ or $x=2$ $x=-\frac{1}{3}$
9.1.3	$f''(x)=-12x+10$ $0=-12x+10$ $12x=10$ $x=\frac{10}{12}=\frac{5}{6}$ $\therefore x<\frac{5}{6}$

9.2.1	
9.2.2	$x < -2$ or $x > 2;\ x \neq 4$

QUESTION 10

10.1	$N(t) = t^3 - 12t^2 + 36t + 8$ $N(0) = 8$ $\therefore$ 8 people
10.2	$N'(t) = 3t^2 - 24t + 36$ increasing $N'(t) \geq 0$ $3t^2 - 24t + 36 \geq 0$ $t^2 - 8t + 12 \geq 0$ $(t-6)(t-2) \geq 0$ $t \geq 6$ or $t \leq 2$ $\therefore$ for first 2 hours after opening or 6 hours after opening until closing time
10.3	Minimum turning point at $t = 6$ hours after opening

QUESTION 11

11.1	$Area = x^2 + 3x^2 + 4xh$ $Area = 4x^2 + 4xh$ $V = x^2h = 1000$ $h = \dfrac{1000}{x^2}$ $A = 4x^2 + 4x\left(\dfrac{1000}{x^2}\right)$ $A = 4x^2 + \dfrac{4000}{x}$

11.2	$A = 4x^2 + 4000x^{-1}$ $A' = 8x - 4000x^{-2} = 0$ $8x = \dfrac{4000}{x^2}$ $x^3 = 500$ $x = \sqrt[3]{500} = 7{,}94\ cm$ $h = \dfrac{1000}{(7{,}94)^2} = 15{,}86\ cm$

PAPER G

QUESTION/*VRAAG* 8

8.1	$f(x+h) = 3 - 2(x+h)^2$ $= 3 - 2x^2 - 4xh - 2h^2$ $f(x+h) - f(x) = 3 - 2x^2 - 4xh - 2h^2 - 3 + 2x^2$ $= -4xh - 2h^2$ $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h) - f(x)}{h}$ $= \lim\limits_{h\to 0}\dfrac{-4xh - 2h^2}{h}$ $= \lim\limits_{h\to 0}\dfrac{h(-4x - 2h)}{h}$ $= \lim\limits_{h\to 0}(-4x - 2h)$ $= -4x$

8.2	$y=\frac{12x^2+2x+1}{6x}$ $=2x+\frac{1}{3}+\frac{1}{6x}$ $=2x+\frac{1}{3}+\frac{1}{6}x^{-1}$ $\frac{dy}{dx}=2-\frac{1}{6}x^{-2}$ $=2-\frac{1}{6x^2}$
8.3	$y=x^3+bx^2+cx-4$ $y'=3x^2+2bx+c$ $y''=6x+2b$ At point of inflection: $y''=6x+2b=0$ Substitute $x=2$: $6(2)+2b=0$ $2b=-12$ $b=-6$ $y=x^3-6x^2+cx-4$ Substitute $(2;4)$: $4=2^3-6(2)^2+c(2)-4$ $2c=24$ $c=12$ $y=x^3-6x^2+12x-4$

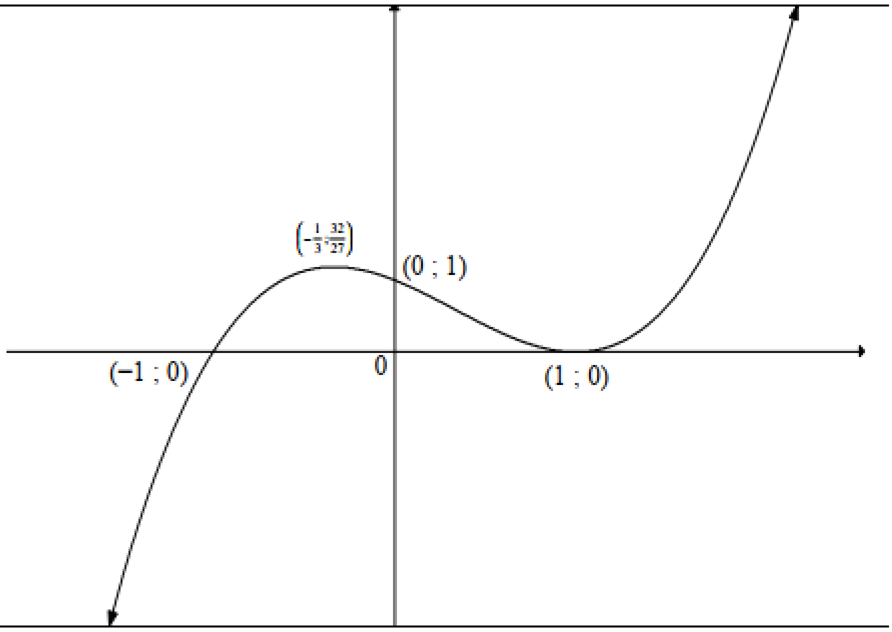
QUESTION/*VRAAG* 9

9.1	(0 ; 1)
9.2	$f(x) = x^3 - x^2 - x + 1$ $f(x) = x^2(x-1) - (x-1)$ $f(x) = (x-1)(x^2-1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (–1; 0); (1; 0)

OR

	$f(x) = x^3 - x^2 - x + 1$ $f(x) = (x-1)(x^2-1)$ $f(x) = (x-1)(x-1)(x+1)$ $f(x) = 0$ $(x-1)(x-1)(x+1) = 0$ x-intercepts: (–1; 0); (1; 0)

9.3	$f(x) = x^3 - x^2 - x + 1$ $f'(x) = 3x^2 - 2x - 1$ $f'(x) = 0$ $(3x+1)(x-1) = 0$ $x = -\frac{1}{3}$ or $x = 1$ $y = \frac{32}{27}$ $\quad y = 0$ $\left(-\frac{1}{3}; \frac{32}{27}\right)$ $\quad (1; 0)$

9.4	
9.5	$f'(x)<0$ $-\frac{1}{3}<x<1$

QUESTION 10

10.1	$s(t)=2t^2-18t+45$ $s'(t)=4t-18$ $s'(0)=4(0)-18$ $\quad=-18\ m/s$
10.2	$s''(t)=4\ \text{m/s}^2$

10.2	$s''(t) = 4\,\text{m/s}^2$
10.3	$4t - 18 = 0$ $4t = 18$ $t = \frac{9}{2}\text{seconds or } 4{,}5\,\text{seconds}$ **OR** $s(t) = 2\left(t - \frac{9}{2}\right)^2 + \frac{9}{2}$ $t = \frac{9}{2}\text{seconds or } 4{,}5\,\text{seconds}$ **OR** $s(t) = 2t^2 - 18t + 45$ $t = -\frac{-18}{2(2)}$ $t = \frac{9}{2}\text{seconds or } 4{,}5\,\text{seconds}$

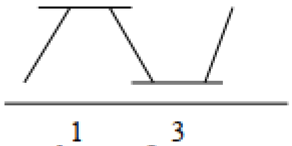
PAPER H

8.1	$f(x+h)=\frac{4}{x+h}$ $f(x+h)-f(x)=\frac{4}{x+h}-\frac{4}{x}$ $=\frac{4x\text{-}4(x+h)}{x(x+h)}$ $=\frac{4x-4x-4h}{x(x+h)}$ $=\frac{-4h}{x(x+h)}$ $\frac{f(x+h)-f(x)}{h}=\frac{\frac{-4h}{x(x+h)}}{h}$ $=\frac{-4h}{xh(x+h)}$ $\frac{-4}{x(x+h)}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-4}{x(x+h)}$ $=\frac{-4}{x^2}$

8.2.1	$y=5x^2+5x+2$ $\frac{dy}{dx}=10x+5$
8.2.2	$D_x\left[\sqrt[3]{x^2}-\frac{1}{2}x\right]$ $=D_x\left[x^{\frac{2}{3}}-\frac{1}{2}x\right]$ $=\frac{2}{3}x^{\frac{-1}{3}}-\frac{1}{2}$

8.3	$p(x)=x^3+2x$ $p'(x)=3x^2+2$ $3x^2\geq 0$ or / *of* $x^2\geq 0$ for all/*vir alle* $x\in \mathbf{R}$ $\therefore 3x^2+2\geq 2>0$ for all/*vir alle* $x\in \mathbf{R}$ i.e. $p'(x)>0$ for all/*vir alle* $x\in \mathbf{R}$ i.e. all tangents to p have gradient greater than (or equal to) 2. Thus there is no tangent to p that has negative gradient.

QUESTION/*VRAAG* 9

9.1	$x=1$ or $x=3$
9.2	$1<x<3$
9.3	For a point x close to 3/*Vir 'n punt naby aan 3*: If $x<3$, $f'(x)<0 \Rightarrow f$ decreasing/*dalend* If $x>3$, $f'(x)>0 \Rightarrow f$ increasing/*stygend* Therefore: f has a local minimum at/*f het lokale minimum by* $x=3$  **OR/*OF*** At $x=3$, the gradient function changes from negative to positive therefore the function will have a local minimum point at $x=3$/ *By* $x=3$ *verander die gradiëntfunksie van negatief na positief dus sal die funksie 'n lokale minimum punt hê by* $x=3$.
	OR/*OF* $f'(3)=0$ and $f''(3)>0$ therefore the function will have a local minimum point at $x=3$ / $f''(3)>0$ *dus sal die funksie 'n lokale minimum punt hê by* $x=3$.
9.4	$f''(x)=0$ at the turning point of/*by die draaipunt van* $f'(x)$ Using symmetry/*Deur simmetrie* $x=\frac{1+3}{2}$ $=2$
9.5	Concave up if/*Konkaaf op as* $f''(x)>0$ $x>2$

QUESTION/*VRAAG* 10

	Given: $M(t) = t^3 - 9t^2 + 3000 \quad ; \quad 0 \le t \le 30$
10.1	$M(0) = 0^3 - 9(0)^2 + 3000$ $= 3000g$ or $3kg$
10.2	$t^3 - 9t^2 + 3000 = 3000$ $t^3 - 9t^2 = 0$ $t^2(t-9) = 0$ $t = 0$ or $t = 9$ Baby's mass will return to the birth mass on the 9th day/ *Baba se massa keer terug na massa by geboorte op die 9de dag.*
10.3	$M'(t) = 0$ $3t^2 - 18t = 0$ $3t(t-6) = 0$ $t = 0$ or $t = 6$ Baby's mass will be a minimum on the 6th day/ *Baba se massa sal 'n minimum wees op die 6de dag.*
10.4	$M'(t) = 3t^2 - 18t$ $M''(t) = 6t - 18$ $0 = 6t - 18$ $t = 3$
	OR / *OF* Using symmetry/*Deur simmitrie*: $t = \frac{0+6}{2}$ $= 3$

PROBABILITY AND COUNTING PRINCIPLES

BASIC PROBABILITY QUESTIONS AND/OR VENN DIAGRAMS

PAPER A

9.1.1	For mutually exclusive events: *Vir onderling uitsluitende gebeurtenisse:* $P(\text{A or B}) = P(\text{A}) + P(\text{B})$ $0,61 = 0,35 + P(\text{B})$ $\therefore P(B) = 0,61 - 0,35$ $= 0,26$
9.1.2	For independent events: *Vir onafhanklike gebeurtenisse:* $P(\text{A or/}\textit{of}\text{ B}) = P(\text{A}) + P(\text{B}) - P(\text{A and/}\textit{en}\text{ B})$ $0,61 = 0,35 + P(\text{B}) - P(\text{A}).P(\text{B})$ $0,61 = 0,35 + P(\text{B}) - 0,35 \times P(\text{B})$ $0,61 = 0,35 + 0,65 \times P(\text{B})$ $\therefore 0,65 \times P(\text{B}) = 0,26$ $\therefore P(\text{B}) = \frac{0,26}{0,65}$ $= 0,4$

PAPER B

QUESTION 4

4.1	P(A or B) = 0,3 + 0,5 = 0,8
4.2	Since A and B are independent P(A or B) = P(A) + P(B) – P(A and B) = 0,3 + 0,5 – 0,15 = 0,65

PAPER C

QUESTION 4

4.1	Since A and C are mutually exclusive, there is no intersection of A and C $\therefore$ P(A and C) = 0.
4.2	Since B and C are independent, P(B and C) = P(B).P(C). P(B and C) = (0,4)(0,2) = 0,08
4.3	Since A and B are independent, P(A and B) = P(A).P(B). P(A and B) = (0,3)(0,4) = 0,12 P(A or B) = P(A) + P(B) – P(A and B) = 0,3 + 0,4 – 0,12 = 0,58

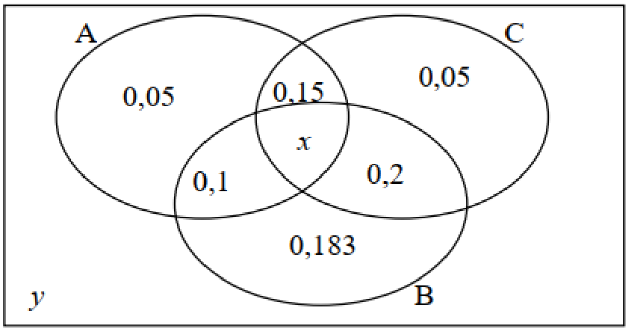
PAPER D

PAPER E

11.1	$P(A) + P(B) = 0{,}52$ $0{,}4 + P(B) = 0{,}52$ $P(B) = 0{,}12$
11.2.1	$P(\text{sandwich}) = \frac{4}{25} = 0{,}16$ **OR/*OF*** $0{,}02 + 0{,}01 + 0{,}04 + 0{,}09 = \frac{4}{25} = 0{,}16$
11.2.2	$P(\text{at least two events}) = 0{,}02 + 0{,}01 + 0{,}03 + 0{,}04$ $= 0{,}1$
11.2.3	$P(\text{not any})$ $= 1 - (0{,}02 + 0{,}01 + 0{,}03 + 0{,}04 + 0{,}04 + 0{,}09 + 0{,}2)$ $= 0{,}57$

PAPER F

QUESTION 10/*VRAAG* 10



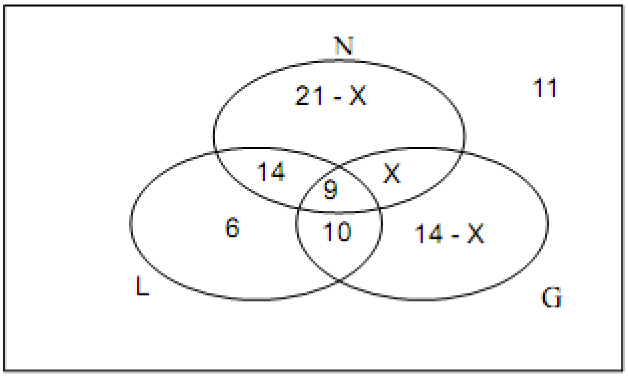
10.1.1(a)	$y = 1 - 0{,}893 = 0{,}107$ (0,11)
10.1.1(b)	$x = 0{,}893 - 0{,}733$ $= 0{,}16$
10.1.2	P(at least 2 events) $= 0{,}1 + 0{,}15 + 0{,}16 + 0{,}2$ $= 0{,}61$ Answer only: Full Marks
10.1.3	P(B) = 0,643 P(C) = 0,56 P(B and C) = 0,36 P(B) × P(C) = 0,643 × 0,56 = 0,36 ∴ P(B and C) = P(B) × P(C) ∴ B and C are independent

PAPER G

QUESTION 4

4.1.1 11 students

4.1.2 Let N represent students reading the *National Geographic* magazine, G represent students reading the *Getaway* magazine and L represent students reading the *Leadership* magazine.



4.1.3 $21 - x + x + 14 - x + 9 + 14 + 10 + 6 + 11 = 80$

$$85 - x = 80$$
$$x = 5$$

4.1.4 P(student reads at least two magazines) $= \dfrac{5+14+10+9}{80} = 0{,}475$

4.2.1

P(smoke detected by device A or device B)

= P(smoke detected by A) + P(smoke detected by B) – P(smoke detected by both)

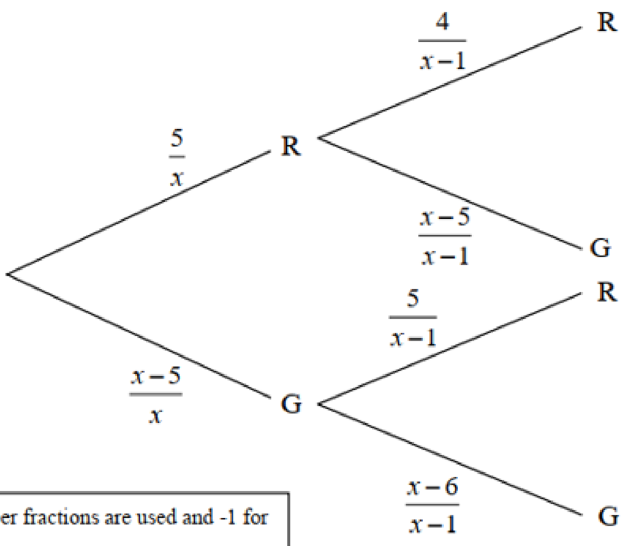
$= 0{,}95 + 0{,}98 - 0{,}94$

$= 0{,}99$

4.2.2 P(smoke not detected) $= 1 - 0{,}99 = 0{,}01$

PAPER H

10.1

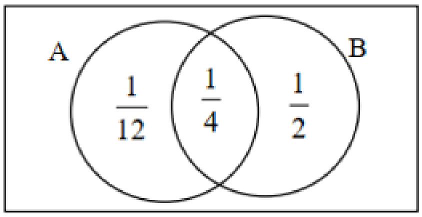
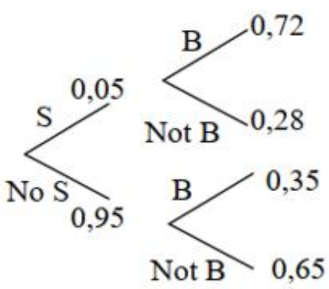


CA only if proper fractions are used and -1 for 2nd pick

VA slegs as egte breuke gebruik word en –1 vir tweede keuse.

$$P(\text{GG}) = P(\text{G}) \times P(\text{G})$$
$$= \frac{x-5}{x} \times \frac{x-6}{x-1}$$
$$\therefore \frac{x-5}{x} \times \frac{x-6}{x-1} = \frac{3}{11}$$
$$11(x-5)(x-6) = 3x(x-1)$$
$$11(x^2 - 11x + 30) = 3x^2 - 3x$$
$$11x^2 - 121x + 330 = 3x^2 - 3x$$
$$8x^2 - 118x + 330 = 0$$
$$4x^2 - 59x + 165 = 0$$

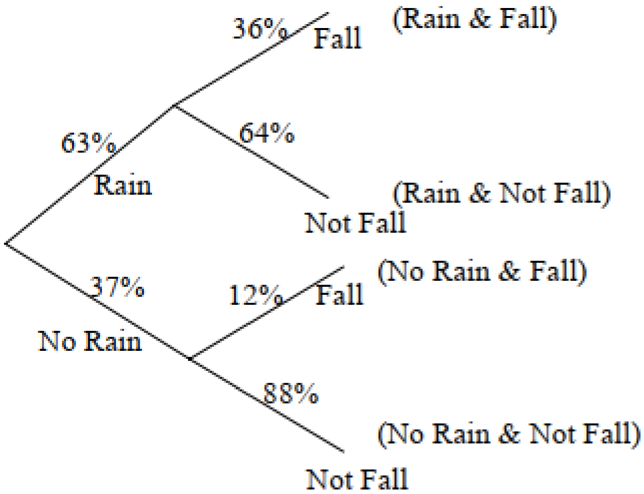
PAPER I

10.1.1	P(A and B) = P(A) × P(B) $= \frac{1}{3} \times \frac{3}{4}$ $= \frac{1}{4}$
10.1.2	P(A or B) = P(A) + P(B) – P(A and B) $= \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$ $= \frac{5}{6}$ **OR/*OF***  P(A or B) = $\frac{1}{12} + \frac{1}{4} + \frac{1}{2} = \frac{5}{6}$
10.2.1	
10.2.2	P(NOT below 0°) = P(S; NOT below 0°) + P(NS ; NOT below 0°) = 0,05 × 0,28 + 0,95 × 0,65 = 0,6315

PAPER J

QUESTION 3

3.1



3.2

$$P(\text{Not Fall}) = \left(\frac{37}{100}\times\frac{88}{100}\right)+\left(\frac{63}{100}\times\frac{64}{100}\right)$$

$$= \frac{407}{1250}+\frac{252}{625}$$

$$= \frac{911}{1250}$$

$$= 0{,}7288$$

3.3

$$P(\text{Dry \& Fall}) = \frac{37}{100}\times\frac{12}{100}$$

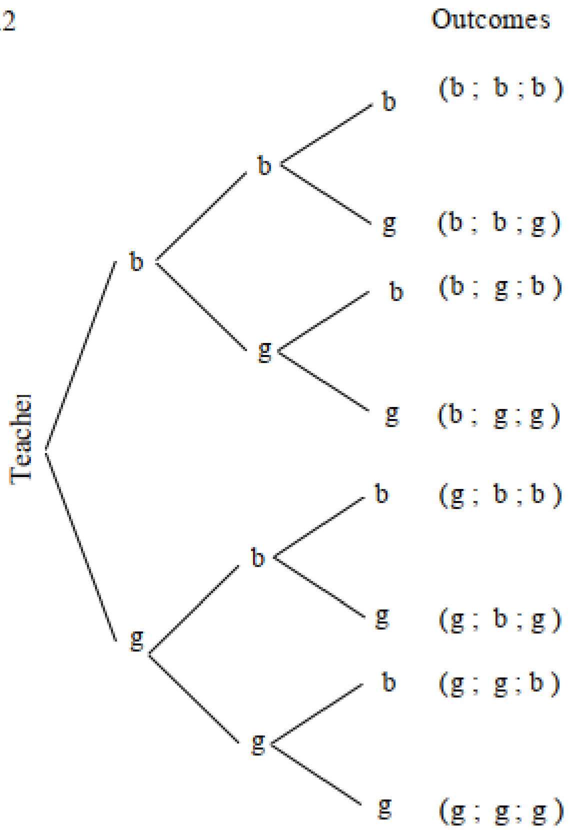
$$= \frac{111}{2500}$$

$$= 0{,}0444$$

PAPER K

4.2.1 $P(\text{boy chosen first}) = \frac{20}{35} = \frac{4}{7} = 0{,}57$.

4.2.2



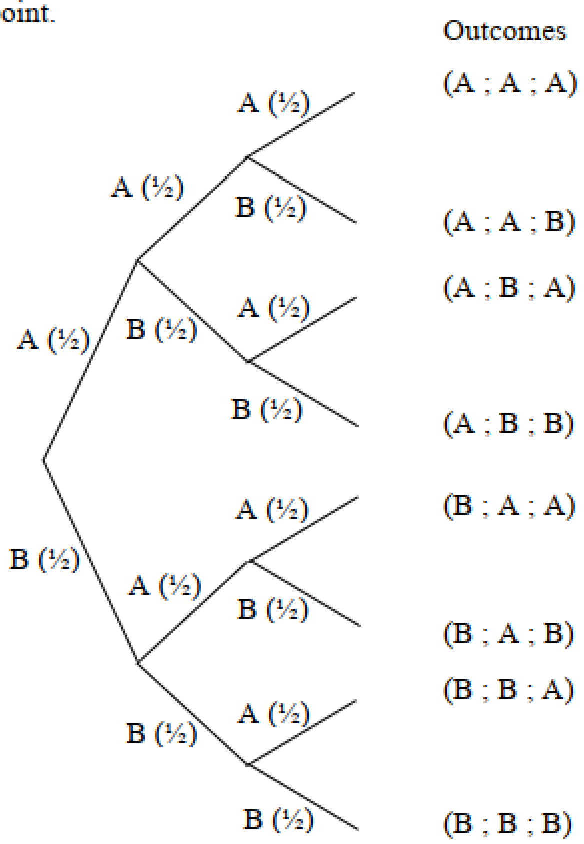
4.2.3 $P(b\,;g\,;b) = \frac{20}{35} \times \frac{15}{34} \times \frac{19}{33} = \frac{190}{1309} = 0{,}15$

4.2.4 $P(g\,;g\,;g) = \frac{15}{35} \times \frac{14}{34} \times \frac{13}{33} = \frac{13}{187} = 0{,}07$

4.2.5 $P(\text{at least one boy}) = 1 - P(\text{three girls chosen})$
$= 1 - 0{,}07$
$= 0{,}93$

PAPER L

5.1 Let A represent Alfred winning a point and B represent Barry winning a point.



5.2 P(Barry wins three points) = $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$

5.3 P(Alfred wins two points and Barry wins one point)

= P(A; A; B) + P(A; B; A) + P(B; A; A)

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{8}$

5.4 P(Alfred wins 3 of the four points)

= P(AAAB) + P(AABA) + P(ABAA) + P(BAAA)

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= 4\left(\frac{1}{2}\right)^4$

$= \frac{1}{4}$

CONTINGENCY TABLES

PAPER M

	<table><tr><td></td><td>Axis Phones</td><td>Direct Phones</td><td>Total</td></tr><tr><td>Defective</td><td>58</td><td>a</td><td>b</td></tr><tr><td>Not Defective</td><td>326</td><td>188</td><td>514</td></tr><tr><td>Total</td><td>384</td><td>c</td><td>600</td></tr></table>
9.2.1	$a=28,\ b=86,\quad c=216$
9.2.2	$\frac{216}{600}=\frac{9}{25}$ or / *of* $0,36$
9.2.3	P(not defective) + P(Axisphones and defective) P(*nie foutief*) + P(*Axis Phones en foutief*) $=\frac{514}{600}+\frac{58}{600}$ $=\frac{572}{600}=\frac{143}{150}$ or/*of* $0,95$

PAPER N

5.1.1	$P(\text{male})=\frac{120}{236}$ $=\frac{30}{59}$ $=0,51\ (0,508474...)$
5.1.2	$P(\text{female and plays sport})$ $=\frac{67}{236}$ $=0,28\ (0,2838983051...)$
5.2	No. From the table, $P(\text{male and do not play sport})=\frac{51}{236}$, which is greater than zero. Since the probability of the intersection of these two events is greater than zero, these events are not mutually exclusive.

5.3	$P(\text{male}) = \frac{120}{236}$ $P(\text{NS}) = \frac{100}{236}$ $P(\text{male}) \times P(NS) = \frac{120}{236} \times \frac{100}{236}$ $= \frac{750}{3\,481}$ $= 0{,}22 \quad (0{,}215455...)$ $P(\text{male and NS}) = \frac{51}{236}$ $= 0{,}22 \quad (0{,}2161016949...)$ So, $P(\text{male}) \times P(NS) = P(\text{male and NS})$ Therefore the events 'male' and 'do not play sport' are independent (correct to TWO decimal places). **OR** The events are not independent as there is a discrepancy from the third decimal place.

PAPER O

5.1	$a = 450$ $b = 319$ $c = 298$ $d = 748$
5.2	P(Female who has not broken a limb) $= \frac{298}{1530}$ $= \frac{149}{765}$
5.3	P(Female & broken a limb) $= \frac{450}{1530}$ $= \frac{5}{17}$ $= 0{,}2941176471...$ $= 0{,}29$

	The events of being female and having broken a limb are independent. If a candidate answers not independent due to the fact that the answers are not accurate to more than 2 decimal places, award full marks.

PROBABILITY AND COUNTING PRINCIPLE

PAPER P

<table>
<tr><td>11.3.1</td><td>$7! = 5040$</td></tr>
<tr><td>11.3.2</td><td>P(4 players alphabetically) $= \frac{1}{7\times6\times5\times4} = \frac{1}{840}$</td></tr>
<tr><td>11.3.3</td><td>
<table><tr><td>F</td><td></td><td>F</td><td></td><td>F</td><td></td><td>F</td></tr></table>
F arrangements: 4!
M arrangements: 5 options with 3 males = $5 \times 4 \times 3$

$4! \times 5 \times 4 \times 3$
$= 1\ 440$

OR/OF

10 Options:
F M F M F M F
M F M F M F F
F F M F M F M
F M F M F F M
M F M F F M F
M F M F F F M
F M F F M F M
M F F M F F M
M F F M F M F
M F F F M F M

Hence $10 \times 4! \times 3! = 1440$
</td></tr>
</table>

PAPER Q

10.3.1	$n(S) = 10!$
10.3.2	4 Options; $\mathbf{2} \times 8 \times 7 \times 6 \times 5 \times 4 \times \mathbf{1} \times 3 \times 2 \times 1 = 80\ 640$ $8 \times \mathbf{2} \times 7 \times 6 \times 5 \times 4 \times 3 \times \mathbf{1} \times 2 \times 1 = 80\ 640$ $8 \times 7 \times \mathbf{2} \times 6 \times 5 \times 4 \times 3 \times 1 \times \mathbf{1} \times 1 = 80\ 640$ $8 \times 7 \times 6 \times \mathbf{2} \times 5 \times 4 \times 3 \times 2 \times 1 \times \mathbf{1} = 80\ 640$ Total number of possibilities = 322 560 $P(\text{5 learners in between}) = \dfrac{322\ 560}{10!} = \dfrac{4}{45}$ **OR/*OF*** $\mathbf{2} \times 8 \times 7 \times 6 \times 5 \times 4 \times \mathbf{1} \times 3 \times 2 \times 1$ 4 possible starting positions $\therefore 4(2 \times 8! \times 1) = 322\ 560$ $8(8!) = 322\ 560$ $P(\text{5 learners in between}) = \dfrac{322\ 560}{10!} = \dfrac{4}{45}$

PAPER R

10.2.1	$7 \times 6 \times 5 \times 4 = 840$
10.2.2	start with 5, 7, 9 or start with 6 or start with 8 $(3 \times 5 \times 1 \times 3) + (1 \times 5 \times 1 \times 2) + (1 \times 5 \times 1 \times 2)$ $= 45 + 10 + 10$ $= 65$ $P = \dfrac{65}{840} = \dfrac{13}{168} = 0{,}08$ **OR/*OF*** ends in 4 or ends in 6 or ends in 8 $(5 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1) + (4 \times 5 \times 1 \times 1)$ $= 25 + 20 + 20$ $= 65$ $P = \dfrac{65}{840} = \dfrac{13}{168} = 0{,}08$

PAPER S

10.2.1	$5!$ $= 120$
10.2.2	5^5 $= 3125$
10.3	$n(\text{E}) = 5! \times 2! \times 2!$ $n(\text{S}) = 7!$ $P(\text{E}) = \dfrac{5! \times 2! \times 2!}{7!}$ $= \dfrac{2}{21}$

PAPER T

QUESTION 7

7.1	Number of ways $= 8 \times 8$ $= 64$
7.2	Number of ways for a 4-digit number $= 8 \times 7 \times 6 \times 5$ $= 1\ 680$ **OR** Number of ways for a 4-digit number $= \dfrac{8!}{(8-4)!}$ $= \dfrac{8!}{4!}$ $= 1680$
7.3	Numbers between 4 000 and 5 000 $= 1 \times 8 \times 8 \times 8$ $= 512$

PAPER U

QUESTION/*VRAAG* 12

12.1.1	$26 \times 25 \times 24 \times 23 \times 22$ $= 7\ 893\ 600$
12.1.2	$24 \times 23 \times 22$ $= 12\ 144$

PAPER V

5.1.1	Number of PIN codes $= 10 \times 10 \times 10 \times 10 \times 10$ $= 10^5$ $= 100\ 000$
5.1.2	Number of PIN codes $= 10 \times 9 \times 8 \times 7 \times 6$ $= 30\ 240$ **OR** Number of PIN codes $= \frac{10!}{5!}$ $= 30\ 240$
5.2	Number of PINs that DO NOT contain 9s $= 9 \times 9 \times 9 \times 9 \times 9$ $= 59\ 049$ P(at least one 9) $= 1 - \text{P(no 9s)}$ $= 1 - \frac{59049}{100000}$ $= 0{,}41$

PAPER W

5.2.1 Any learner seated in any position in: $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 720$ different ways.

5.2.2 $2 \times 5! = 240$

PAPER X

QUESTION 6

6.1	Number of different ways the shirts and trousers can be arranged $=(7+4)!$ $=11!$ $=39\ 916\ 800$
6.2	Number of ways so that the shirts are together and trousers are together $=7!.4!.2$ $=241\ 920$
6.3	P(Shirt at beginning and trouser at the end) $=\dfrac{9!\times 4\times 7}{11!}$ $=\dfrac{14}{55}$

PAPER Y

QUESTION 5

5.1	Number of arrangements $=7!$ $=5040$
5.2	Number of arrangements $=5!$ $=120$
5.3	Number of arrangements $=3!\times 5!$ $=720$

PAPER Z

QUESTION/*VRAAG* 11

11.1	$8\times 7\times 6\times 5\times 4$ or $\dfrac{8!}{3!}$ $=6720$

6.1 $S=\frac{10!}{2!2!}=907\ 200$

Starting with M and ending with M:

$\frac{8!}{2!}=20\ 160$

Starting with A and ending with A:

$\frac{8!}{2!}=20\ 160$

The probability of starting and ending with the same letter is: $\frac{2(20160)}{907\ 200} = 0.04$

PAPER 2

STATISTICS

PAPER A

QUESTION 1

1.1.1	$\bar{y} = \frac{155}{10}$ $= 15{,}5$
1.1.2	SD $= 4{,}59$
1.2	$\bar{y} - \text{SD}$ $= 15{,}5 - 4{,}59$ $= 10{,}91$ $\therefore\ 10 - 2 = 8$ learners
1.3	$a = 1{,}7709\ldots$ $b = 0{,}2243\ldots$ $\hat{y} = 1{,}77 + 0{,}22x$
1.4	$\hat{y} = 1{,}77 + 0{,}22(72)$ $= 17{,}61$ ≈ 18 votes **OR/*OF*** $\hat{y} = 17{,}92 \approx 18$ votes
1.5.1	Points are all scattered therefore low correlation and unrealistic prediction./*Punte is versprei daarom 'n lae korrelasie en onrealistiese voorspelling.*
1.5.2	$r = 0{,}98$/correlation very strong/*korrelasie baie sterk* $\therefore$ a reliable prediction/*'n betroubare voorspelling*

QUESTION 2

2.1	60 employees
2.2	$20 < x \leq 25$
2.3	60 – 34 = 26 employees
2.4	Salary $= \frac{100}{7} \times 2400$ Salary = R34 285,71
2.5	∴ Ogive/Cumulative frequency graph will shift to the right/will become steeper. ∴ *Ogief/Kumulatiewe frekwensie grafiek sal na regs skuif/sal steiler wees.*

PAPER B

QUESTION 1

1.1	$a = -23{,}846\ldots$ $b = 0{,}227\ldots$ $\hat{y} = -23{,}85 + 0{,}23x$
1.2	$\hat{y} = -23{,}85 + 0{,}23(550)$ $y = 102{,}65$ **OR** $y = 101{,}02$
1.3	$r = 0{,}98$
1.4	Very strong positive correlation

50	100	130	150	180	190	200	200

1.5.1	$\bar{x} = \frac{1200}{8}$ $\bar{x} = 150$ **OR** $\bar{x} = 150$
1.5.2	$\sigma = 50,50$
1.5.3	$\bar{x} - \sigma$ $= 150 - 50,50$ $= 99,50$ $\therefore$ 1 stop

QUESTION 2

2.1

Number of glasses of water per day	Number of staff members	Cumulative frequency
$0 \leq x < 2$	5	5
$2 \leq x < 4$	15	20
$4 \leq x < 6$	13	33
$6 \leq x < 8$	5	38
$8 \leq x < 10$	2	40

2.2	40 staff members
2.3	33 staff members

2.4	$\bar{x} = \dfrac{\left(1\times\left(5+\dfrac{k}{2}\right)\right)+(3\times15)+\left(5\times\left(13+\dfrac{k}{2}\right)\right)+(7\times5)+(9\times2)}{40+k}=4$ $5+\dfrac{k}{2}+45+65+\dfrac{5k}{2}+35+18=160+4k$ $3k+168=160+4k$ $k=8$ **OR** $\bar{x} = \dfrac{(1\times5)+(15\times3)+(13\times5)+(5\times7)+(2\times9)}{40}$ $=4{,}2$ $\bar{x}_{\text{old}}-\bar{x}_{\text{current}}=4{,}2-4$ $=0{,}2$ $\therefore 0{,}2\times40$ $=8$ teachers

PAPER C

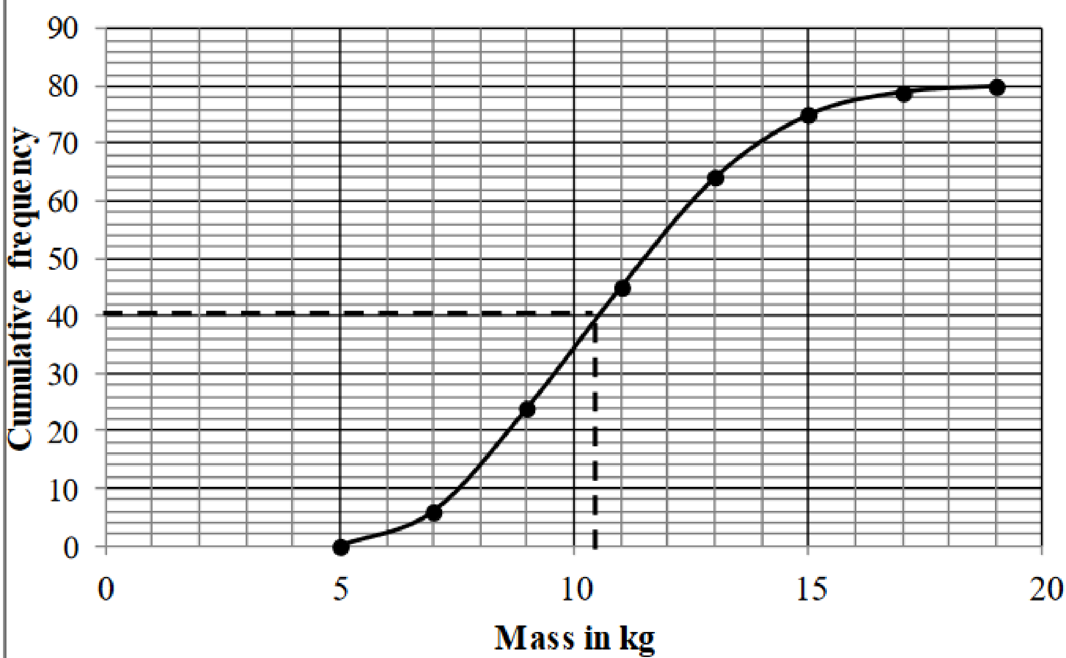
QUESTION/*VRAAG* 1

1.1 Modal class: $9 < m \leq 11$

1.2

Mass (in kg)	Frequency	Cumulative frequency
$5 < m \leq 7$	6	6
$7 < m \leq 9$	18	24
$9 < m \leq 11$	21	45
$11 < m \leq 13$	19	64
$13 < m \leq 15$	11	75
$15 < m \leq 17$	4	79
$17 < m \leq 19$	1	80

1.3



1.4 Median mass: 10,5 kg

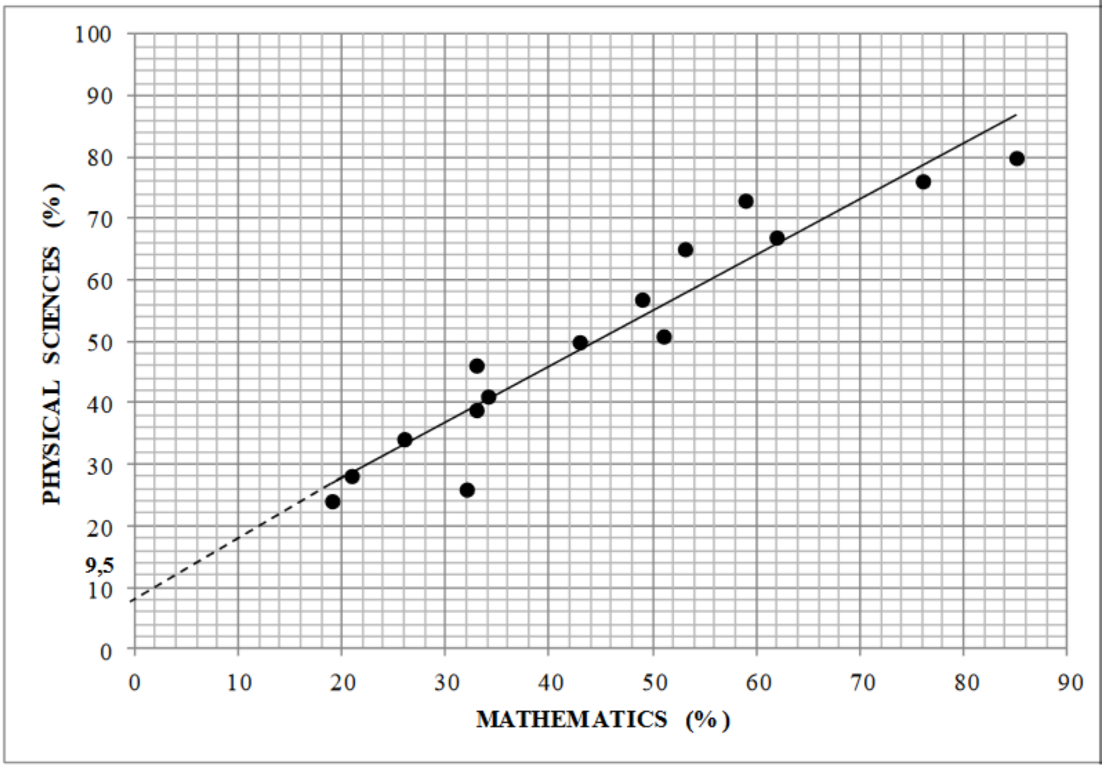
1.5.1	$\bar{x} = \dfrac{(6\times6+18\times8+21\times10+19\times12+11\times14+4\times16+1\times18)}{80}$ $= \dfrac{854}{80}$ $= 10{,}68$
1.5.2	Learners' bags are heavier than the stipulated international guideline. Estimated mean = 10,68 kg 10% of 80 kg = 8 kg 10,68 kg > 8 kg
	OR/ *OF* Learners' bags are heavier than the stipulated international guideline. Estimated mean $= \dfrac{10,68}{80}\times100$ $= 13{,}35\%$ 13,35% > 10%

QUESTION 2

2.1	$a = 634{,}382\ldots$ $b = 32\ 189{,}263\ldots$ $\hat{y} = 634{,}38 + 32189{,}26x$
2.2	$\hat{y} = 634{,}38 + 32189{,}26(0{,}25)$ $= \text{R}8\ 681{,}70$ **OR/*OF*** $\hat{y} = \text{R}8\ 681{,}70$ (if using calculator)
2.3	Average price increase $= \text{R}\dfrac{32189{,}26}{20}$ per 0,05 carat $= \text{R}1\ 609{,}46$ per 0,05 carat **OR/*OF*** Average price increase $= 0{,}05 \times 32\,189{,}26$ $= \text{R}1\ 609{,}46$ per 0,05 carat **OR/*OF*** at 0,3: $\hat{y} = \text{R}10\,291{,}16$ $\therefore$ Average price increase $= 10\ 291{,}16 - 8\ 681{,}70$ $= \text{R}1\ 609\ 46$ per 0,05 carat
2.4	The point (0,35 ; 11500) is closer to the least squares regression line.

PAPER D

QUESTION 1

1.1	$a = 9{,}5$ $b = 0{,}909.. = 0{,}91$ $\hat{y} = 9{,}5 + 0{,}91x$
1.2	
1.3	Final exam mark $\approx$ 72,22% (calculator) **OR** $\hat{y} = 9{,}5 + 0{,}91(69)$ $\approx 72{,}29\%$
1.4	$r = 0{,}95$
1.5	There is a **very strong positive** correlation between the Mathematics and Physical Sciences mark.

1.6	The teacher concludes that the higher the learners' Mathematics marks, the higher the learners' Physical Sciences marks.

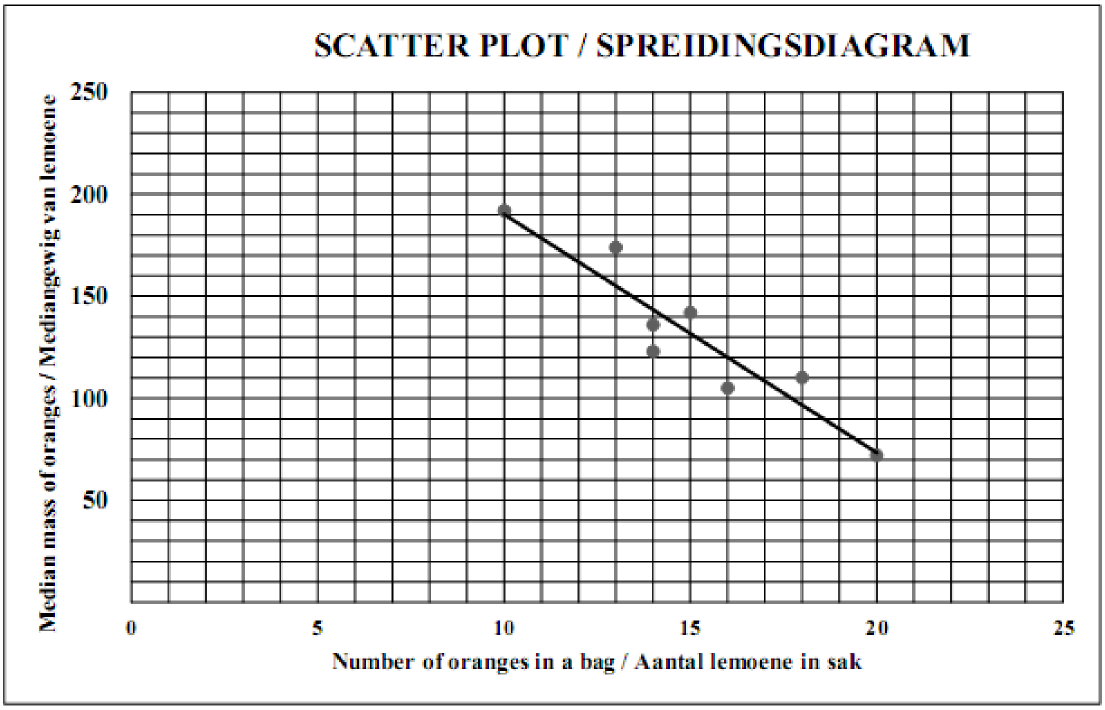
QUESTION 2

2 018	**2 175**	**2 182**	**2 215**	**2 254**	**2 263**	**2 267**	**2 271**	**2 293**	**2 323**	**2 334**	**2 346**

2.1	July / *Julie*
2.2	$\bar{x} = \dfrac{26941}{12}$ $= 2\ 245{,}083.. \approx 2\ 245{,}08$ aircraft landings
2.3	Standard deviation for landings at the King Shaka International airport: $\sigma = 86{,}30$
2.4	$(\bar{x} - \sigma\,;\bar{x} + \sigma) = (2\ 245{,}08 - 86{,}30\ ;\ 2\ 245{,}08 + 86{,}30)$ limit $= (2\ 158{,}78\ ;\ 2\ 331{,}38)$ There were **9** months when the aircraft arrivals at the King Shaka International airport were within one standard deviation of the mean.
2.5	The standard deviation of the number of landings at the Port Elizabeth Airport will be higher than the standard deviation of the number of arrivals at the King Shaka International Airport **OR C.**

PAPER E

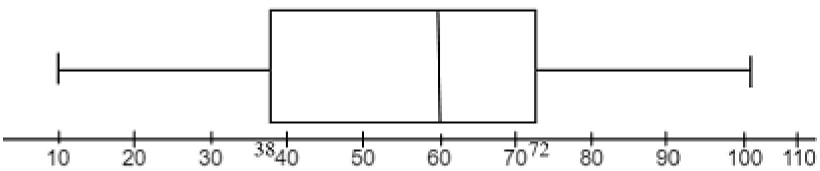
QUESTION 1



1.1	$a = 307,20$ $b = -11,70$ $\hat{y} = 307,20 - 11,7x$
1.2	$r = -0,93$

1.3	See scatter plot above/*sien spreidingsdiagram hierbo* (10 ; 190,2) (20 ; 73,2)
1.4	Negative strong association / *Negatiewe sterk assosiasie*
1.5	$\hat{y} = 307,20 - 11,7(12)$ $= 166,8$

QUESTION / *VRAAG* 2

2.1.1	100
2.1.2	Median / *Mediaan* = ±62
2.1.3	
2.1.4	Skewed to the left / *Skeef na links*
2.2	$b = 20$ $\frac{d-a}{2} = 8$ $2a = d$ $sub\ \frac{2a-a}{2} = 8$ $a = 16$ $d = 32$ $5 + 16 + 19 + 20 + c + 32 + 35 = 7 \times 22$ $\therefore c = 27$

PAPER F

QUESTION 1

1.1	$\bar{x} = \dfrac{1\,581}{31}$ $= 51$ **OR / *OF*** $\bar{x} = 51$ (calculator method / *sakrekenaar metode*)
1.2	$\therefore$ skewed to the left
1.3	Physical Sciences performed better. Q_1 is 40% in Physical Sciences and 28% in Mathematics which indicates the lower 25% of the class performed much better in Physical Sciences than in Mathematics. *Fisiese Wetenskappe presteer beter.* *Q_1 is 40% in Fisiese Wetenskappe en 28% in Wiskunde wat aandui dat die onderste 25% van die klas heelwat beter presteer in Fisiese Wetenskappe as in Wiskunde.*
1.4	Accept any mark between 40 – 50. *Aanvaar enige punt tussen 40 – 50 .*
1.5	The greatest difference is $87\% - 71\% = 16\%$

QUESTION / *VRAAG* 2

2.1	$a = 12{,}41$ $b = 0{,}49$ $\hat{y} = 12{,}41 + 0{,}49x$
2.2	$\hat{y} = 12{,}41 + 0{,}49x$ $= 12{,}41 + 0{,}49(150)$ $= 85{,}91 \approx 86\%$ **OR/OF** $\hat{y} = 85{,}17$
2.3	$\hat{y} = 12{,}41 + 0{,}49x$ The y-intercept is 12,41 which means that a learner who did not begin the exam achieved 12,41%. This is clearly impossible.

2.4	10,28
2.5	$63{,}9-\sigma=p$ $63{,}9+\sigma=103{,}59$ $127{,}92=p+103{,}59$ $p=24{,}33$ **OR / *OF*** $\sigma=103{,}59-63{,}96$ $=39{,}63$ $p=63{,}96-39{,}63$ $=24{,}33$

PAPER G

QUESTION 1

1.1.1(a)	$\bar{x}=\frac{375}{15}$ $\bar{x}=25\text{ MB}$
1.1.1(b)	$\sigma=17{,}65\text{ MB}$
1.1.2	$25+17{,}65=42{,}65$ $\therefore 2$ days
1.1.3	Overall $\bar{x}=\frac{80}{100}\times 25$ $=20\text{ MB}$ $\frac{375+x}{30}=20$ $x=600-375$ $=225$ maximum total amount of data that Sam must use for the remainder of the month: **225** MB

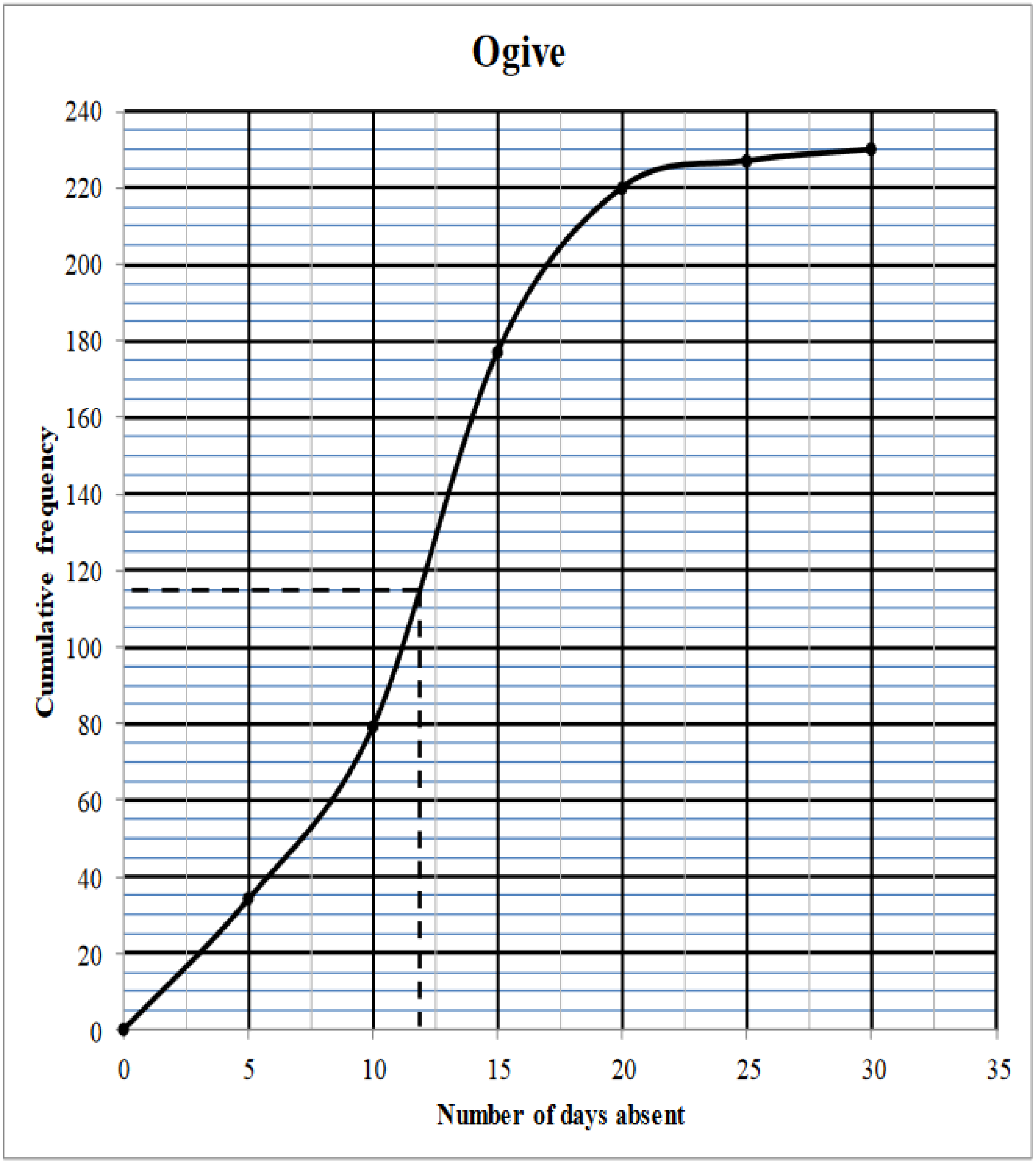
1.2.1	$a = 29,35$ $b = -0,46$ $\hat{y} = 29,35 - 0,46x$
1.2.2	$y = 25,20$ °C (calculator) OR $\hat{y} = 29,35 - 0,46(9)$ $y = 25,21$ °C
1.2.3	$b < 0$, indicating that as the wind speed increases the temperature decreases.

QUESTION/VRAAG 2

Number of days absent	Number of learners	Cumulative frequency
$0 \leq x < 5$	34	34
$5 \leq x < 10$	45	79
$10 \leq x < 15$	98	177
$15 \leq x < 20$	43	220
$20 \leq x < 25$	7	227
$25 \leq x < 30$	3	230

2.1	Modal class: $10 \leq x < 15$
2.2	177 learners
2.3	230 learners

2.4



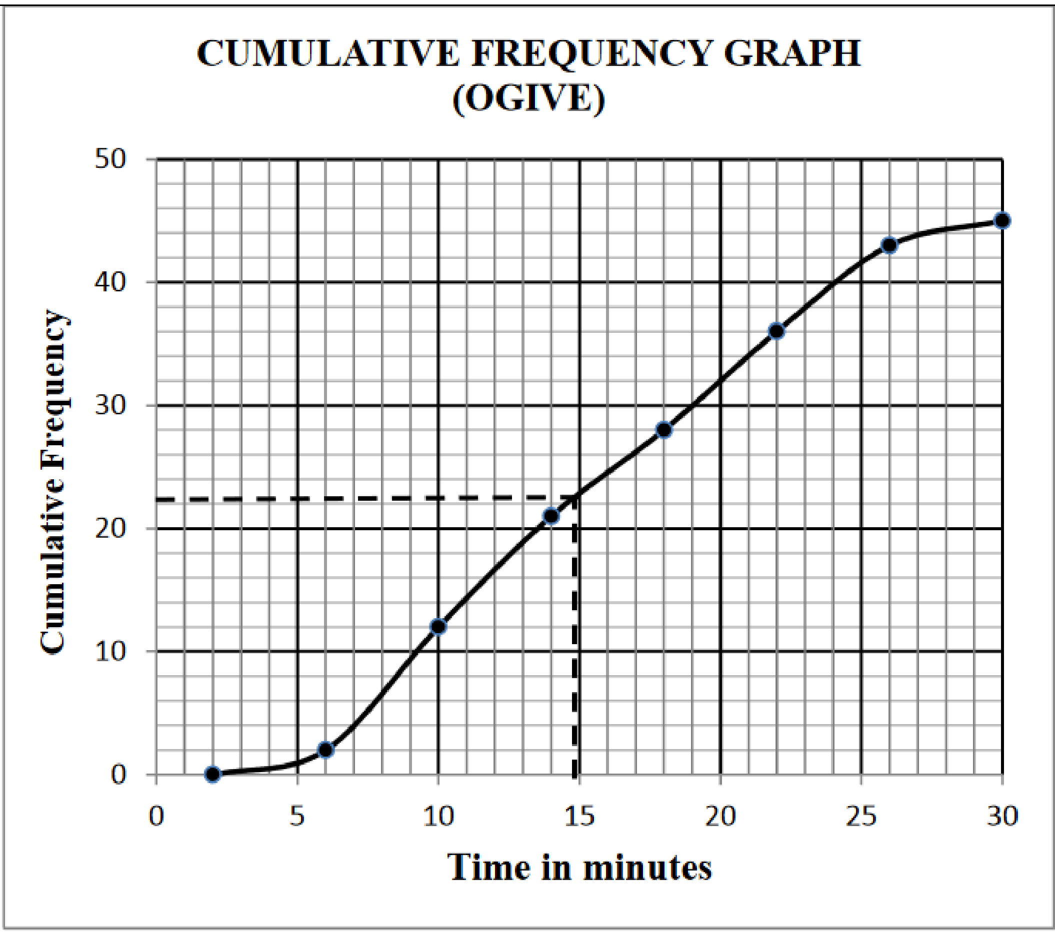
2.5 The median is at position 115.
□value of median is 12 days (accept 11 – 14)

PAPER H

QUESTION/*VRAAG* 1

1.1	45 children
1.2	$\bar{x} = \frac{\sum fx}{n} = \frac{(4\times 2)+(8\times 10)+(12\times 9)+(16\times 7)+(20\times 8)+(24\times 7)+(28\times 2)}{45}$ $\bar{x} = \frac{692}{45}$ OR $\bar{x} = 15{,}38$ minutes
1.3	

Time taken (t) (in minutes)	Number of children	Cumulative frequency
$2 < t \leq 6$	2	2
$6 < t \leq 10$	10	12
$10 < t \leq 14$	9	21
$14 < t \leq 18$	7	28
$18 < t \leq 22$	8	36
$22 < t \leq 26$	7	43
$26 < t \leq 30$	2	45

1.4	
1.5	On graph at the y-value of 22,5 or 23 Median = $\pm$ 15 minutes.

QUESTION/*VRAAG* 2

2.1	$a = 12{,}44$ $b = 0{,}98$ $y = 12{,}44 + 0{,}98x$
2.2.1	$\text{Percentage} = \frac{15}{50} \times 100$ $= 30\%$
2.2.2	$\hat{y} = 12{,}44 + 0{,}98x$ $\hat{y} = 12{,}44 + 0{,}98(30)$ $\hat{y} = 41{,}84$ $= 42$ **OR** $\hat{y} = 41{,}87$ (if using calculator) $\hat{y} = 42$ **OR** $\hat{y} = \frac{21}{50}$
2.3.1	standard deviation $= 13{,}88$
2.3.2	$x = 50{,}67 - 45{,}67$ $= 5\%$

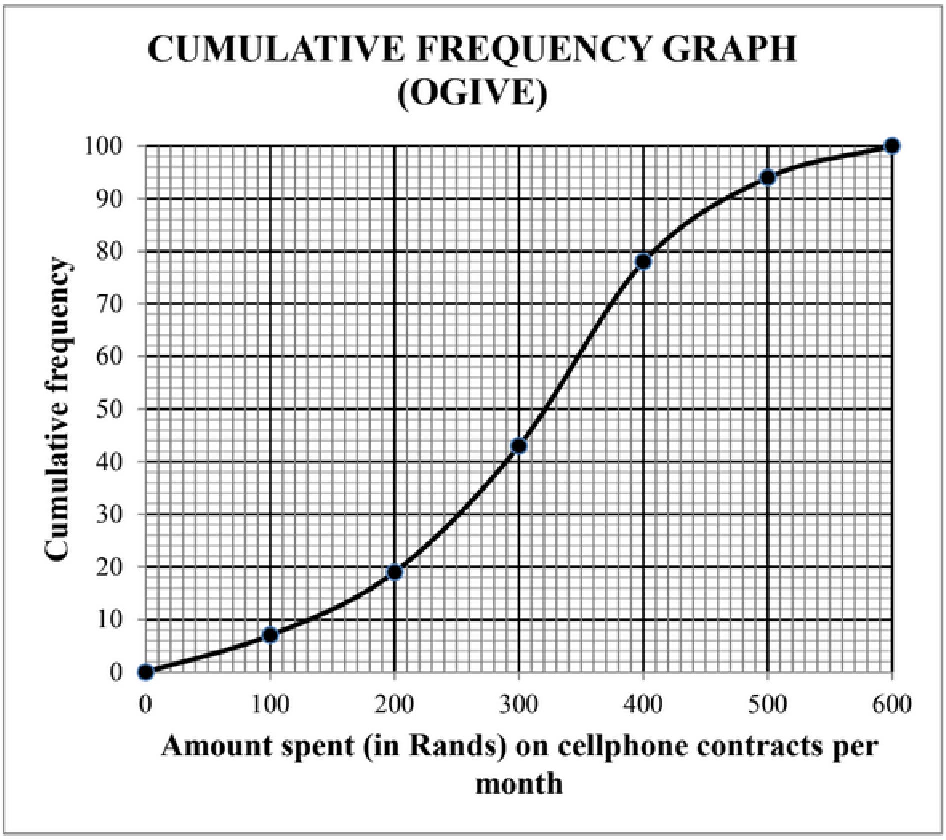
PAPER I

QUESTION 1

1.1	$a = -1946{,}875... = -1946{,}88$ $b = 0{,}41$ $\hat{y} = -1946{,}88 + 0{,}41x$
1.2	Monthly repayment $\approx$ R3 727,16 (calculator) *Maandelikse paaiement* $\approx$ R3 727,16 **OR** $\hat{y} = -1946{,}88 + 0{,}41(14000)$ $\approx$ R3 793,12
1.3	$r = 0{,}946 \ldots \approx 0{,}95$
1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. **OR** D

QUESTION/*VRAAG* 2

2.1	Number people paid R200 or less = 19 *Aantal mense wat R200 of minder betaal het = 19*
2.2	$7 + 12 + a + 35 + b + 6 = 100$ $a = 40 - b$ $309 = \dfrac{(50\times 7)+(150\times 12)+(250\times a)+(350\times 35)+(450\times b)+(550\times 6)}{100}$ $309 = \dfrac{(50\times 7)+(150\times 12)+(250\times (40-b))+(350\times 35)+(450\times b)+(550\times 6)}{100}$ $350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$ $200b = 3200$ $b = 16$ $a = 24$

	OR/*OF* $7+12+a+35+b+6=100$ $b=40-a$ $309=\dfrac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$ $309=\dfrac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times(40-a))+(550\times6)}{100}$ $350+1800+250a+12250+1800-450a=30900$ $200a=4\ 800$ $a=24$ $b=16$
2.3	Modal class/*modale klas:* $300<x\leq400$
2.4	
2.5	Number of people/*Aantal mense* = 100 – 82 [accept 80 – 84 people] 18 people paid more than R420 per month/. [accept 16 – 20 people]

PAPER J

QUESTION/*VRAAG 1*

1.1.1	$a = 1730{,}22$ $b = 13{,}96$ $\hat{y} = 1730{,}22 + 13{,}96x$
1.1.2	$\hat{y} = 1730{,}22 + 13{,}96x$ $\hat{y} = 1730{,}22 + 13{,}96(28\,500)$ $\hat{y} = \text{R}399\ 590{,}22$ **OR/*OF*** $\hat{y} = \text{R}399\ 599{,}64$ (calc)
1.1.3	$r = 0{,}98002\ \ldots$ $r = 0{,}98$
1.1.4	There is a very strong positive correlation between the amount spent on advertising and sales. /

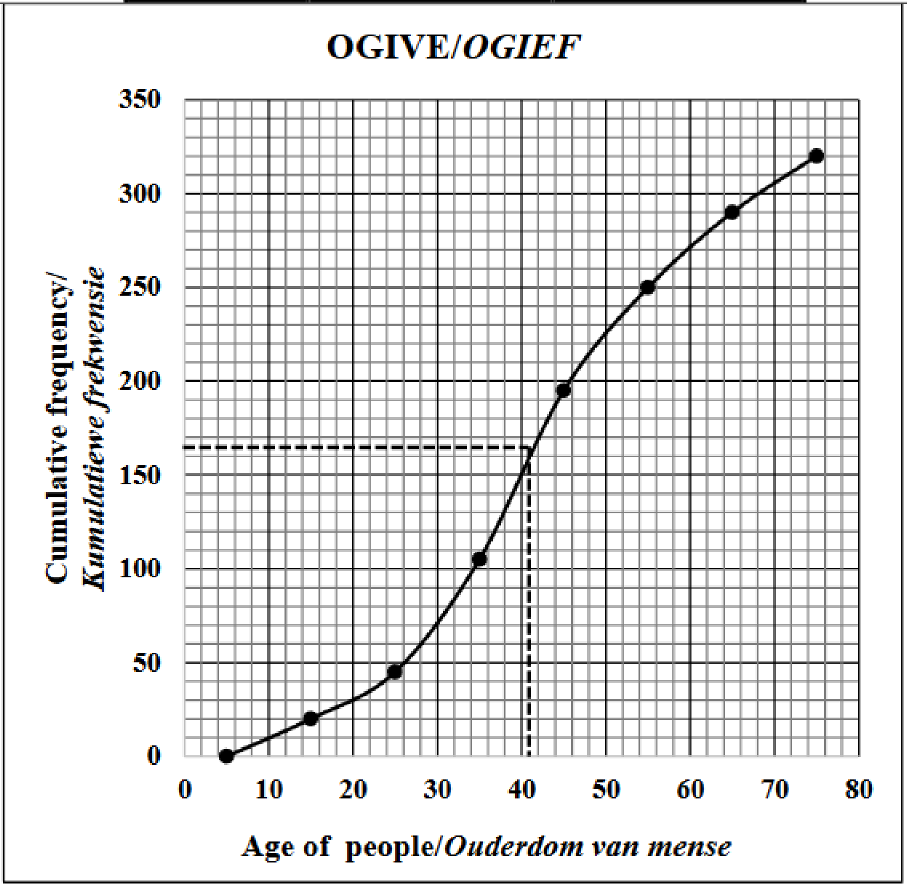
1.2.1	$\bar{x} = \dfrac{1\,552\,195}{9}$ $\bar{x} = 172\,466{,}11$
1.2.2	$\sigma = 56\,950{,}09$
1.2.3	$\bar{x} + \sigma$ $= 172\,466{,}11 + 56\,950{,}09$ $= 229\,416{,}20$ 2 years/*jaar*

QUESTION/*VRAAG* 2

2.1	$35 < x \leq 45$
2.2	320 people/*mense*

2.3

AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY
$5 < x \leq 15$	20	20
$15 < x \leq 25$	25	45
$25 < x \leq 35$	60	105
$35 < x \leq 45$	90	195
$45 < x \leq 55$	55	250
$55 < x \leq 65$	40	290
$65 < x \leq 75$	30	320



2.4	Median = 41

ANALYTICAL GEOMETRY

PAPER A

QUESTION 3

3.1.1	$m_{AB}=\dfrac{2-(-4)}{4-6}$ **OR** $m_{AB}=\dfrac{-4-2}{6-4}$ $m_{AB}=-3$
3.1.2	$\tan\alpha=m_{AB}=-3$ $\alpha=108{,}43°$
3.1.3	$T\left(\dfrac{x_1+x_2}{2};\dfrac{y_1+y_2}{2}\right)$ $T\left(\dfrac{-2+6}{2};\dfrac{-3-4}{2}\right)$ $T\left(2;\dfrac{-7}{2}\right)$
3.1.4	$5(0)-6y=8$ $y=-\dfrac{4}{3}$ $S\left(0;\dfrac{-4}{3}\right)$
3.2	$m_{CD}=m_{AB}=-3$ $-3=-3(-2)+c$ OR $y-(-3)=-3(x-(-2))$ $c=-9$ $\quad$ $y=-3x-9$ $y=-3x-9$

3.3.1	$5x-6y=8$ $y=\dfrac{5}{6}x-\dfrac{8}{6}$ $\tan\theta=m_{AC}=\dfrac{5}{6}$ $\theta=39{,}81°$ $\hat{A}=108{,}43°-39{,}81°$ $=68{,}62°$ $D\hat{C}A=68{,}62°$ [alt ∠s ; DC‖AB]

3.3.2 $P(-3;0)$ and $F(1,6\ ;\ 0)$

Area POSC = Area ΔFPC – Area ΔOFS

$$=\frac{1}{2}(4,6)(3)-\frac{1}{2}(1,6)\left(\frac{4}{3}\right)$$

$$=6,9-1,07$$

$$=5,83\text{ units}^2$$

OR/*OF*

$P(-3;0)$

$$FC=\sqrt{\left(-2-\frac{8}{5}\right)^2+(-3-0)^2}=\frac{3\sqrt{61}}{5}$$

$$\text{Area }\Delta PFC=\frac{1}{2}(PF)(FC)\sin O\hat{F}S$$

$$=\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39,81°$$

$$=6,90$$

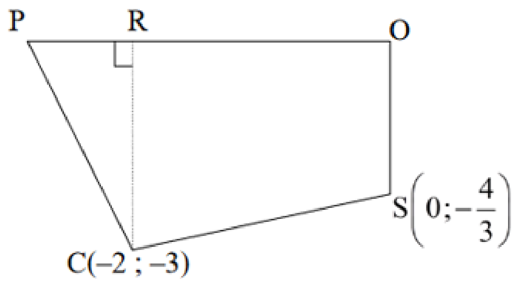
$$\text{Area }\Delta OFS=\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$$

$$=1,07$$

Area POSC $=6,90-1,07$

$=5,83\text{ units}^2$

OR/*OF*



$P(-3;0)$

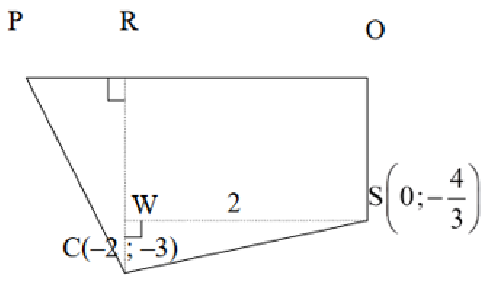
Area of POSC = Area of OSCR + Area of ΔPRC

$$=\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2+\frac{1}{2}(1\times 3)$$

$$=\frac{35}{6}$$

$$=5,83\text{ units}^2$$

OR/
OF



$P(-3;0)$

Area POSC = Area ROSW + Area ΔPRC + Area ΔWSC

$$=\left(\frac{4}{3}\right)(2)+\frac{1}{2}(1)(3)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$$

$$=\frac{35}{6}$$

$$=5{,}83\text{units}^2$$

OR/*OF*

$P(-3;0)$

$$\text{Area of }\Delta PSC=\frac{1}{2}(PC)(CS)\sin D\hat{C}A$$

$$=\frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68{,}62°$$

$$=3{,}833..$$

$$\text{Area of }\Delta POS=\frac{1}{2}(PO)(OS)$$

$$=\frac{1}{2}(3)\left(\frac{4}{3}\right)$$

$$=2$$

Area POSC $= 3{,}833\ldots + 2$

$$=5{,}83\text{units}^2$$

QUESTION 4

4.1	$P(x;y)$; $N(7;-2)$; $M(3;-5)$ $\frac{x+7}{2}=3$ $\quad\quad$ $\frac{y-2}{2}=-5$ $x=-1$ $\quad\quad$ $y=-8$ $P(-1;-8)$
4.2.1	$r^2=(7-3)^2+(-2-(-5))^2$ **OR/OF** $r^2=(-1-3)^2+(-8-(-5))^2$ $r^2=25$ $(x-3)^2+(y+5)^2=25$
4.2.2	$m_{\text{radius}}=\frac{-5-(-2)}{3-7}=\frac{3}{4}$ $m_{\text{tangent}}=-\frac{4}{3}$ [radius $\perp$ tangent/*raaklyn* $\perp$ *radius*] $-2=-\frac{4}{3}(7)+c$ $\quad$ **OR** $\quad$ $y-(-2)=-\frac{4}{3}(x-7)$ $c=\frac{22}{3}$ $\quad\quad$ $y=-\frac{4}{3}x+\frac{22}{3}$ $y=-\frac{4}{3}x+\frac{22}{3}$
4.3	$-8=-\frac{4}{3}(-1)+c$ $\therefore\ c=-\frac{28}{3}$ $-\frac{28}{3}<k<\frac{22}{3}$

4.4.1	$AB^2=AM^2-MB^2$ $AB^2=\left[(t-3)^2+(t+5)^2\right]-5^2$ $=t^2-6t+9+t^2+10t+25-25$ $AB=\sqrt{2t^2+4t+9}$
4.4.2	$t=\frac{-4}{2(2)}$ $=-1$ Minimum at $t=-1$ $AB=\sqrt{2(-1)^2+4(-1)+9}$ $AB=\sqrt{7}$

	OR/*OF* $4t+4=0$ $t=-1$ Minimum at $t=-1$ $AB=\sqrt{2(-1)^2+4(-1)+9}$ $AB=\sqrt{7}$ **OR/*OF*** Length of $AB=\sqrt{2t^2+4t+9}$ $=\sqrt{2\left(t^2+2t+\frac{9}{2}\right)}$ $=\sqrt{2\left[(t+1)^2+\frac{7}{2}\right]}$ $=\sqrt{2(t+1)^2+7}$ Minimum at $t=-1$ $AB=\sqrt{2(-1)^2+4(-1)+9}$ $AB=\sqrt{7}$

PAPER B

QUESTION 3

3.1.1	$y=-x-11$ $A(-1\,;\,t)$ $t=-(-1)-11$ $t=-10$
3.1.2	$\tan\alpha=-1$ $ref.\angle=45°$ $\therefore \alpha=135°$
3.1.3	$\tan 63{,}43°=m_{AC}$ $m_{AC}=2$
3.2	$m_{AC}=2$ $A(-1\,;\,-10)$ $y=2x+k$ **OR/*OF*** $y-y_1=2(x-x_1)$ $-10=2(-1)+k$ $y-(-10)=2(x-(-1))$ $k=-8$ $y=2x-8$ $y=2x-8$

3.3.1	$y = 2x - 8$ $0 = 2x - 8$ $x_B = 4$ $\frac{x_C + (-1)}{2} = 4$ $\quad$ $\frac{y_C + (-10)}{2} = 0$ $x_C = 9$ $\quad$ $y_C = 10$ **OR/*OF*** by translation / *met translasie* $A \rightarrow B\ (x;y) \rightarrow (x+5; y+10)$ $B \rightarrow C\ (4;0) \rightarrow (4+5;\ 0+10) = (9\ ;\ 10)$
3.3.2	$A\hat{B}E = 63,43°$ [vert. opp $\angle$'s =] $\hat{E}_2 = 63,43°$ [corres. $\angle$'s, DE || AB] $\hat{E}_1 = 45°$ [$\angle$s on a str line] $F\hat{E}D = 108,43°$ **OR/*OF*** $E\hat{A}B = 135° - 63,43°$ $E\hat{A}B = 71,57°$ $D\hat{E}A = E\hat{A}B = 71,57°$ $F\hat{E}D = 108,43°$ **OR/*OF*** $A\hat{B}E = 63,43°$ [vert. opp $\angle$'s] $D\hat{E}O = 116,57°$ [co-int. $\angle$'s, DE || AB] $F\hat{E}D = 360° - (116,57° + 135°)$ $= 108,43°$
3.4	$y = 0$ $x_E = -11$ $\frac{x_G + (-11)}{2} = 4$ $x_G = 19$ $(x-19)^2 + y^2 = 15^2$ $(x-19)^2 + y^2 = 225$

QUESTION 4

4.1	$M(-6;-3)$
4.2.1	$x^2+y^2+24x-10y+153=0$ $(x+12)^2+(y-5)^2=-153+144+25$ $(x+12)^2+(y-5)^2=16$ $r^2=16$ $r=4$ units
4.2.2	$NM=\sqrt{(-12-(-6))^2+(5-(-3))^2}$ $NM=10$ units $SM=5$ units $\therefore TS=10-5-4\;=1$ unit
4.3.1	$R(-6\,;-8)$ $y=-8$

4.3.2	$m_{NM}=\dfrac{5-(-3)}{-12-(-6)}$ $m_{NM}=-\dfrac{4}{3}$ $m_{tangent}=\dfrac{3}{4}$ $-5=\dfrac{3}{4}(-17)+c$ **OR/*OF*** $y-y_1=\dfrac{3}{4}(x-x_1)$ $c=\dfrac{31}{4}$ $\quad$ $y-(-5)=\dfrac{3}{4}(x-(-17))$ $y=\dfrac{3}{4}x+\dfrac{31}{4}$ $\quad$ $y=\dfrac{3}{4}x+\dfrac{31}{4}$

	OR/OF NS = SM = 5 $S\left(\frac{-12-6}{2}\ ;\ \frac{5-3}{2}\right)$ $S(-9\ ;\ 1)$ $m_{SK}=\frac{1-(-5)}{-9+17}$ $=\frac{6}{8}=\frac{3}{4}$ $y+5=\frac{3}{4}(x+17)$ $y=\frac{3}{4}x+\frac{31}{4}\quad or\quad y=\frac{3}{4}x+7\frac{3}{4}$
4.4.1	$-8=\frac{3}{4}x+\frac{31}{4}$ $-32=3x+31$ $3x=-63$ $x=-21$ P(–21 ; –8) R(–6 ; –8) PR = PS = 15 units [tangents from same point] MS = MR = 5 units Perimeter PSMR = 15 + 15 + 5 + 5 = 40 units

4.4.2	$\dfrac{\text{area of }\Delta NPS}{\text{area of quadrilateral }PSMR}$ $\dfrac{\frac{1}{2}NS.SP}{\frac{1}{2}SP.MS+\frac{1}{2}MR.PR}$ $=\dfrac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $=\dfrac{1}{2}$ **OR** $\Delta NPS \equiv \Delta SPM \equiv \Delta MPR$ $\dfrac{\text{area of }\Delta NPS}{\text{area of quadrilateral }PSMR}$ $=\dfrac{1}{2}$

PAPER C

3.1	$SL=\sqrt{(x_S-x_L)^2+(y_S-y_L)^2}$ $SL=\sqrt{(4-(-4))^2+(5-1)^2}$ $SL=\sqrt{80}=4\sqrt{5}=8,94\text{ units}$
3.2	$m_{SN}=\dfrac{5-(-3)}{4-(-2)}$ $m_{SN}=\dfrac{4}{3}$
3.3	$m=\tan\theta=\dfrac{4}{3}$ $\theta=53,13°$
3.4	$m_{LN}=\dfrac{1-(-3)}{-4-(-2)}$ $m_{LN}=-2$ $L\hat{K}O=116,565...°$ $L\hat{N}S=116,565...°-53,13°$ $L\hat{N}S=63,44°$

	OR $SN=10\text{ units}$ $\sin L\hat{N}S=\dfrac{4\sqrt{5}}{10}$ $L\hat{N}S=63,44°$

	OR $\text{LN} = 2\sqrt{5}$ units $\tan \text{L}\hat{\text{N}}\text{S} = \dfrac{4\sqrt{5}}{2\sqrt{5}}$ $\text{L}\hat{\text{N}}\text{S} = 63{,}44°$ **OR** $\text{SN} = 10$ units $\text{LN} = 2\sqrt{5}$ units $\cos \text{L}\hat{\text{N}}\text{S} = \dfrac{2\sqrt{5}}{10}$ $\text{L}\hat{\text{N}}\text{S} = 63{,}44°$
3.5	$m = \dfrac{4}{3}$ $1 = \dfrac{4}{3}(-4) + c$ **OR** $y - 1 = \dfrac{4}{3}(x - (-4))$ $c = \dfrac{19}{3}$ $y - 1 = \dfrac{4}{3}x + \dfrac{16}{3}$ $y = \dfrac{4}{3}x + \dfrac{19}{3}$ $y = \dfrac{4}{3}x + \dfrac{19}{3}$
3.6	$\text{SL} = 4\sqrt{5}$ $\text{LN} = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$ $\text{LN} = \sqrt{20} = 2\sqrt{5}$ $\text{Area } \Delta\text{LSN} = \dfrac{1}{2}(4\sqrt{5})(2\sqrt{5})$ $= 20 \text{ units}^2$ **OR**

	SN = 10 units $LN=\sqrt{(-4-(-2))^2+(1-(-3))^2}$ $LN=\sqrt{20}=2\sqrt{5}$ $\text{Area } \Delta LSN=\frac{1}{2}(10)\left(2\sqrt{5}\right)\sin 63,44°$ $=20 \text{ units}^2$
3.7	$\hat{L}=90°$ SN is a diameter of circle S, L, N [chord subtends 90° **OR** converse ∠ in semi-circle] $\text{Centre of circle}=P\left(\frac{4+(-2)}{2};\frac{5+(-3)}{2}\right)$ $=P(1;1)$ **OR** Let the coordinates of P be (a ; b). Then, PL = PN: $(-4-a)^2+(1-b)^2=(-2-a)^2+(-3-b)^2$ $a-2b=-1$ …..equation 1 If PS = PN, then: $4a+2b=6$ ………….. equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1;1)$ **OR** If PL = PN, then: $a-2b=-1$ …..equation 1 If PS = PL, then: $2a+b=3$ …..equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1;1)$

3.8	$\hat{LPN} = \theta = 53{,}13°$ [alt ∠s; LP ∥ x –axis] $\therefore \hat{LPS} = 126{,}87°$ **OR** $\hat{LNS} = 63{,}44°$ $\therefore \hat{LPS} = 126{,}88°$ [∠ at centre = 2 ×∠ at circumference] **OR** $\hat{LSN} = 26{,}56°$ [sum of ∠s in Δ] $\hat{SLP} = 26{,}56°$ [∠s opp equal radii] $\therefore \hat{LPS} = 126{,}88°$ [sum of ∠s in Δ] **OR** $(4\sqrt{5})^2 = 5^2 + 5^2 - 2(5)(5)\cos \hat{LPS}$ $\cos \hat{LPS} = -\frac{3}{5}$ $\therefore \hat{LPS} = 126{,}87°$

PAPER D

3.1	$E\left(\frac{12}{2};\frac{6}{2}\right)$ $E(6;3)$
3.2	$m_{BA} = \frac{6-0}{7-5}$ $=3$ $y = mx + c$ $y = 3x + c$ $6 = 3(7) + c$ **OR / *OF*** $y - y_1 = m(x - x_1)$ $c = -15$ $y - 6 = 3(x-7)$ $y = 3x - 15$ $y = 3x - 21 + 6$ $y = 3x - 15$

3.3	$rx - 3y + 5 = 0$ $-3y = -rx - 5$ $y = \frac{r}{3}x + \frac{5}{3}$ $3 = \frac{r}{3}$ $r = 9$

3.4	Area Δ AOP $=10$ $\frac{1}{2}\times \text{AO}\times \perp h=10$ $\frac{1}{2}\times 5\times \perp h=10$ $\perp h=4$ but / *maar* $y<0$ $\therefore y=-4$ $\text{AP}=\text{BP}$ $\text{AP}^2=\text{BP}^2$ $(x-5)^2+(-4-0)^2=(x-7)^2+(-4-6)^2$ $x^2-10x+25+16=x^2-14x+49+100$ $4x=108$ $x=27$ P $(27\,;-4)$

Question 4

4.1	$\alpha=135°$ ext $\angle$ of Δ / *buite* $\angle$ *van* Δ $\tan(135°)=m$ $m=-1$ $y=mx+c$ $y=-1x+c$ $4=-1(-2)+c$ $c=2$ $y=-x+2$ **OR / *OF*** $y-y_1=m(x-x_1)$ $y-4=-1(x+2)$ $y=-1x-2+4$ $y=-x+2$

4.2	$\text{P}(-4;0)$ $a=-4$ $m_{\text{BA}}.m_{\text{AS}}=-1$ $m_{\text{BA}}=1$ $m_{\text{BA}}=\frac{4-b}{-2+4}$ $1=\frac{4-b}{2}$ $2=4-b$ $b=2$

4.3	$(x-a)^2+(y-b)^2=r^2$ $(x+4)^2+(y-2)^2=r^2$ $(-2+4)^2+(4-2)^2=r^2$ $4+4=r^2$ $(x+4)^2+(y-2)^2=8$
4.4	$x^2-2x+y^2-2y=0$ $(x^2-2x+1)+(y^2-2y+1)=1+1$ $(x-1)^2+(y-1)=2$
4.5	D (1 ; 1)
4.6	$\text{DE} = \sqrt{2}$ $\text{DA}=\sqrt{(-2-1)^2+(4-1)^2}$ $=\sqrt{9+9}$ $=\sqrt{18}$ **OR/*OF*** $=3\sqrt{2}$ $\text{D}\hat{\text{E}}\text{A}=90°$ radius $\perp$ tangent $\text{AD}^2=\text{DE}^2+\text{AE}^2$ pythagoras $\left(\sqrt{18}\right)^2=\left(\sqrt{2}\right)^2+\text{AE}^2$ $18-2=\text{AE}^2$ $\text{AE}=4$

PAPER E

QUESTION 3

3.1	$m_{AB} = \dfrac{3 - \frac{1}{2}}{5 - 0}$ $m_{AB} = \dfrac{1}{2}$
3.2	$m_{CE} = m_{BA} = \dfrac{1}{2}$ $-4 = \dfrac{1}{2}(6) + c$ **OR/*OF*** $y - (-4) = \dfrac{1}{2}(x - 6)$ $c = -7$ $y = \dfrac{1}{2}x - 7$
3.3.1	D(0 ; –7) $\dfrac{x_C + 6}{2} = 0$ $\dfrac{y_C + (-4)}{2} = -7$ $x_C = -6$ $y_C = -10$ C(–6 ; –10)

3.3.2	$\text{Area } \Delta\text{BCD} = \dfrac{1}{2}(7{,}5)(6)$ $= 22{,}5$ $\text{Area } \Delta\text{ABD} = \dfrac{1}{2}(7{,}5)(5)$ $= 18{,}75$ $\text{Area ABCD} = 22{,}5 + 18{,}75 = 41{,}25 \text{ units}^2$

3.4.1	K(–6 ; –4)
3.4.2a	KC = 6 units; KE = 12 units; $CE = \sqrt{(6)^2 + (12)^2}$ [Pythagoras] $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ Perimeter $\Delta KEC = 6 + 12 + \sqrt{180}$ $= 31,42$ units
3.4.2b	$\tan K\hat{C}E = \frac{KE}{KC} = \frac{12}{6} = 2$ $K\hat{C}E = 63,43°$ **OR/*OF*** $\sin K\hat{C}E = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $K\hat{C}E = 63,43°$ **OR/*OF*** $m_{CE} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26,57°$ $K\hat{C}E = 90° - 26,57°$ $K\hat{C}E = 63,43°$

OR/*OF*

$$KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos K\hat{C}E$$

$$(12)^2 = (6)^2 + \left(\sqrt{180}\right)^2 - 2(6)\left(\sqrt{180}\right)(\cos K\hat{C}E)$$

$$\cos K\hat{C}E = \frac{\sqrt{5}}{5}$$

$$K\hat{C}E = 63{,}43°$$

QUESTION 4

4.1.1	$y = x + 1$ $b = a + 1$
4.1.2	$MR^2 = MK^2$ $(a-6)^2 + (b-0)^2 = (a-5)^2 + (b-7)^2$ $(a-6)^2 + (a+1)^2 = (a-5)^2 + (a+1-7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore M(2\ ;\ 3)$
4.2.1	$(6-2)^2 + (0-3)^2 = r^2$ $r = 5$ **OR/*OF*** $(2-5)^2 + (3-7)^2 = r^2$ $r = 5$

4.2.2	T(–2 ; 0) TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** M(2 ; 3) F(a ; 0) FR = 4 units TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** $(x-2)^2+(0-3)^2=25$ $x^2-4x+4+9=25$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$ or $x=-2$ TR = 8 units
4.3	$m_{\text{radius}}=\frac{7-3}{5-2}$ $m_{\text{radius}}=\frac{4}{3}$ $m_{\text{tangent}}=-\frac{3}{4}$ $7=-\frac{3}{4}(5)+c$ **OR/*OF*** $y-7=-\frac{3}{4}(x-5)$ $c=\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$
4.4.1	N(2 ; –2)
4.4.2	$(-2-2)^2+(0+2)^2=r^2$ $r^2=20$ $(x-2)^2+(y+2)^2=20$

PAPER F

3.1.1	$1 = \frac{3+x}{2}$ $\quad$ $-2 = \frac{4+y}{2}$ $2 = 3 + x$ $\quad$ $-4 = 4 + y$ $x = -1$ $\quad$ $y = -8$ $B(-1;-8)$
3.1.2	$m_{CD} = \frac{0-4}{6-3}$ $= -\frac{4}{3}$
3.1.3	$y - 2 = \frac{-4}{3}(x - 11)$ $y = \frac{-4}{3}x + \frac{50}{3}$ **OR / *OF*** $y = \frac{-4}{3}x + c$ $2 = \frac{-4}{3}(11) + c$ $c = \frac{50}{3}$ $y = \frac{-4}{3}x + \frac{50}{3}$

3.1.4	$CD = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ $= \sqrt{(0-4)^2 + (6-3)^2}$ $= \sqrt{25}$ $CD = 5$ **OR / *OF*** $R(5;10)$ midpoint / *middelpun* $RQ = \sqrt{(2-10)^2 + (11-5)^2}$ $RQ = 10$ D is the midpoint of PR / *D is die middelpunt van PR* C is the midpoint of PQ (line from midpoint of 1 side || to 2nd side)/ *C is die middelpunt van PQ (lyn van middelpunt van 1 sy || aan 2de sy)* $RQ = 2CD = 10$ (midpoint theorem/ *middelpuntstelling*) $PK = RQ$ $\sqrt{(y+2)^2 + (4-1)^2} = 10$ $(y+2)^2 + (4-1)^2 = 10^2$ $(y+2)^2 = 91$ $y+2 = \pm\sqrt{91}$ $y = \pm\sqrt{91} - 2$ $y = -11{,}54$ or / *of* $y \neq 7{,}54$ **or / *of*** $\sqrt{(y+2)^2 + (4-1)^2} = 10$ $y^2 + 4y + 4 + 9 = 100$ $y^2 + 4y - 87 = 0$ $y = \frac{-4 \pm \sqrt{4^2 - 4(1)(-87)}}{2(1)}$ $y = \frac{-4 \pm \sqrt{364}}{2}$ $y = -11{,}54$ **or / *of*** $y \neq 7{,}54$

3.2.1	$m_{PQ} = \tan\theta$ $\tan\theta = 1$ $\theta = 45°$ $\hat{P}_1 = 35°$ vertical opp $\angle$s / *regoorst* $\angle e$ QR || to the x-axis / *aan die x-as* $\hat{T}_1 = 35° + 45°$ ext $\angle$ of Δ / *buite* $\angle$ v Δ $\hat{T}_1 = 80°$ $\alpha = \hat{T}_1 = 80°$ corr $\angle$s ST||QR / *ooreenkomstige* $\angle e$ *ST||QR*

3.2.2	x at/*by* U: $\dfrac{U_2+(-8)}{2}=1$ $\therefore U_x = 10$ units / *eenhede* QU = 18 units / *eenhede* x at/*by* W = x at / *by* U = 10 y at/*by* W: $y = 10+\dfrac{2}{3}$ $=\dfrac{32}{3}$ $\text{WU} = \dfrac{32}{3}+5=\dfrac{47}{3}$ $\therefore \text{Area } \Delta\text{QWU} = \dfrac{1}{2}(18)\left(\dfrac{47}{3}\right)$ $=141$ square units / *eenhede kwadraat*

Question 4

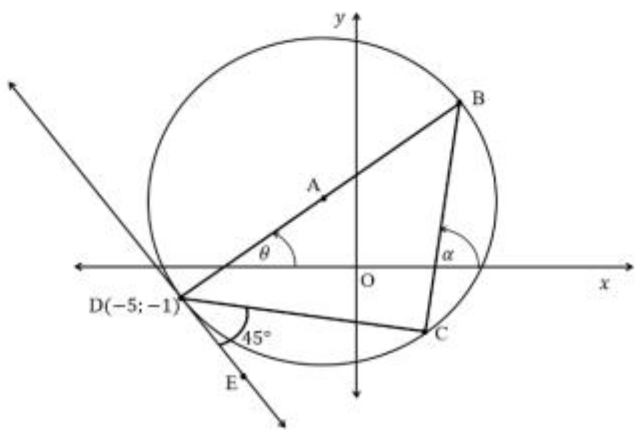
4.1	$\text{T}\hat{\text{U}}\text{S} = 180° - 101{,}31° = 78{,}69°$ adj supp ∠s / *aangrensende suppl* ∠e $m_{\text{TU}} = \tan 78{,}69° = 5$ $c = 6$ $y = 5x+6$
4.2	$x-\text{int}$ / *afsnit* $y = 0$ $\dfrac{-1}{5}x+\dfrac{4}{5}=0$ $-x+4=0$ $x=4$ $\therefore \text{S}(4;0)$ $\text{M} = \left(\dfrac{-6+4}{2};\dfrac{2+0}{2}\right)$ $\therefore \text{M}(-1;1)$

4.3	$(x+1)^2+(y-1)^2=r^2$ $(-6+1)^2+(2-1)^2=r^2$ $r^2=26$ $(x+1)^2+(y-1)^2=26$ **OR / *OF*** $(x+1)^2+(y-1)^2=r^2$ $(4+1)^2+(0-1)^2=r^2$ $r^2=26$ $(x+1)^2+(y-1)^2=26$
4.4	$m_{MP}=-\frac{1}{5}$ $\quad m_{MP}\times m_{KL}=-1$ $m_{KL}=5$ $\quad$ radius $\perp$ tan / *radius $\perp$ raaklyn* $m_{TU}=5$ $\quad$ proven / *reeds bewys* $\therefore m_{TU}=m_{KL}=5$ KL || TU
4.5	$\mathrm{VM}=\sqrt{\left(-1+\frac{1}{2}\right)^2+(1-7)^2}$ $=6{,}02$ radius $=\sqrt{26}=5{,}1$ $6{,}02>5{,}1$ $\therefore \mathrm{V}\left(-\frac{1}{2};7\right)$ does not lie within the circle / *lê nie binne die sirkel nie.*

PAPER G

3.1	AC=BD $\sqrt{50} = \sqrt{(3-(-2))^2 + (p-(-4))^2}$ $50 = 25 + p^2 + 8p + 16$ $p^2 + 8p - 9 = 0$ $(p+9)(p-1) = 0$ $p \neq -9\,; p = 1$
3.2	$M\left(\frac{3-2}{2};\frac{1-4}{2}\right)$ $M\left(\frac{1}{2};-\frac{3}{2}\right)$
3.3	$m_{DC} = \frac{1-(-2)}{3-4}$ $m_{DC} = \frac{3}{-1}$ $m_{DC} = -3$
3.4	$y = -3x + c$ $-4 = -3(-2) + c$ $-10 = c$ $y = -3x - 10$

QUESTION 4



4.1.1	$A(-1;2)$	

4.1.2	$\frac{x+(-5)}{2}=-1$ $x-5=-2$ $x=3$ $\frac{y+(-1)}{2}=2$ $y-1=4$ $y=5$ $B(3;5)$
4.1.3	$m_{AD}=\frac{2-(-1)}{-1-(-5)}$ $=\frac{3}{4}$
4.1.4	$tan\theta=\frac{3}{4}$ $\theta=36{,}87°$

4.1.5	$m_{radius}=\frac{3}{4}$ $m_{tangent}=\frac{-4}{3}$ $\quad radius \perp tangent$ $y=-\frac{4}{3}x+c$ $-1=-\frac{4}{3}(-5)+c$ $-1=\frac{20}{3}+c$ $c=-\frac{23}{3}$ $\therefore y=-\frac{4}{3}x-\frac{23}{3}$

4.2	$\hat{B} = 45°$ [tan;chord theorem / raaklyn;koord stelling] $\alpha = 45° + 36{,}87°$ $[ext \angle of\ \Delta/buite\angle van\Delta]$ $\alpha = 81{,}87°$ $m_{BC} = \tan 81{,}87°$ $m_{BC} = 7$
4.3.1	$x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$ $(x-3)^2 + (y+1)^2 = 18$ $\therefore$ M $(3;-1)$
4.3.2	M$(3;-1)$ and A $(-1;2)$ MA $= \sqrt{(3-(-1))^2 + (-1-2)^2}$ $MA = \sqrt{16+9}$ $MA = \sqrt{25}$ $MA = 5$ $r_M + r_A = \sqrt{18} + 5$ or $= 9{,}24$ MA< $r_M + r_A$ $\therefore$ *circles intersect/sirkels sny*

PAPER H

QUESTION 3

3.1	$\mathrm{M}\left(\frac{4+8}{2};\frac{-8+0}{2}\right)$ $\mathrm{M}(6;-4)$
3.2	$m_{\mathrm{NS}}=\frac{0-(-16)}{8-0}$ or $m_{\mathrm{NQ}}=\frac{0-(-8)}{8-4}$ or $m_{\mathrm{QS}}=\frac{-8-(-16)}{4-0}$ $=2$
3.3	$m_{\mathrm{LQ}}\times 2=-1$ [LQ $\perp$ NS] $\therefore m_{\mathrm{LQ}}=-\frac{1}{2}$ $-8=-\frac{1}{2}(4)+c$ **OR** $y+8=-\frac{1}{2}(x-4)$ $c=-6$ $y+8=-\frac{1}{2}x+2$ $\therefore y=-\frac{1}{2}x-6$
3.4	OS is the radius of circle passing through S $(x-0)^2+(y-0)^2=(16)^2$ $x^2+y^2=256$
3.5	$m_{\mathrm{RM}}=m_{\mathrm{LQ}}=-\frac{1}{2}$ [RM $\parallel$ LQ] $-4=-\frac{1}{2}(6)+c$ **OR** $y+4=-\frac{1}{2}(x-6)$ $c=-1$ $y+4=-\frac{1}{2}x+3$ $\therefore y=-\frac{1}{2}x-1$ $\mathrm{T}(0;-1)$

3.6	$T(0;-1)$, $P(0;-6)$ and $S(0;-16)$ $\therefore$ PS = 10 units and TS = 15 units $\frac{LS}{RS}=\frac{PS}{TS}=\frac{2}{3}$ [prop theorem; RM || LP] **OR** [line || one side of Δ/*lvn* || *een sy v Δ*] **OR** M(6 ; –4), Q(4 ; –8) and S(0 ; –16) $MS=\sqrt{180}=6\sqrt{5}$ and $QS=\sqrt{80}=4\sqrt{5}$ $\frac{LS}{RS}=\frac{QS}{MS}=\frac{2}{3}$ [prop theorem; RM || LQ] **OR** [line || one side of Δ/*lyn* || *een sy v Δ*]
3.7	area of PTMQ = area of ΔTSM – area of ΔPSQ $=\frac{1}{2}.ST.\perp h_M-\frac{1}{2}.PS.\perp h_Q$ $=\frac{1}{2}(15)(6)-\frac{1}{2}(10)(4)$ $=45-20$ $=25$ square units
	OR $TM=\sqrt{45}=3\sqrt{5}=6{,}71$ $MQ=\sqrt{20}=2\sqrt{5}=4{,}47$ $PQ=\sqrt{20}=2\sqrt{5}=4{,}47$ area of trapezium PTMQ $=\frac{1}{2}\left(3\sqrt{5}+2\sqrt{5}\right)\left(2\sqrt{5}\right)$ $=\frac{1}{2}\left(5\sqrt{5}\right)\left(2\sqrt{5}\right)$ $=25$ square units

	OR $MQ = \sqrt{20} = 2\sqrt{5}$ $PQ = \sqrt{20} = 2\sqrt{5}$ $TP = 5$ area of PTMQ= area of ΔMTP+ area of ΔPQM $\boxed{\text{area of PTMQ} = \frac{1}{2}TP \times \perp h_M + \frac{1}{2}MQ \times PQ}$ area of PTMQ$= 10 + 15 = 25$

QUESTION 4

4.1	$PV = r = \sqrt{10}$ $PV = \sqrt{(k-(-3))^2 + (1-4)^2} = \sqrt{10}$ $(PV)^2 = (k-(-3))^2 + (1-4)^2 = 10$ $k^2 + 6k + 9 + 9 = 10$ **OR** $(k+3)^2 + 9 = 10$ $k^2 + 6k + 8 = 0$ $(k+3)^2 = 1$ $(k+4)(k+2) = 0$ $k+3 = 1$ or $k+3 = -1$ $k = -4$ or $k = -2$ $\therefore k = -2$
4.2	$x^2 + 6x + y^2 - 8y + 15 = 0$ y-intercepts: $(0)^2 + 6(0) + y^2 - 8y + 15 = 0$ $(y-3)(y-5) = 0$ $y_C = 3$ or $y_B = 5$ $\therefore BC = 2$ units

4.3.1	$m_{TC} = \dfrac{3-1}{0-(-2)}$ $= 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$ **OR** $y = mx + 3$ $1 = m(-2) + 3$ $m_{TC} = 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$
4.3.2	$B\hat{C}V = 135°$ [ext $\angle$ of Δ/*buite* $\angle$ v Δ] $\therefore V\hat{W}B = 45°$ [opp $\angle$s of cyclic quad/*teenoorst.* $\angle$*e v kvh*] **OR** $T\hat{C}O = 45°$ [$\angle$s of Δ/$\angle$*e v* Δ] $\therefore V\hat{W}B = 45°$ [ext $\angle$s of cyclic quad/*buite* $\angle$ *v kvh*]
4.4.1	$Q(-3;-2)$
4.4.2	$(x+3)^2 + (y+2)^2 = 10$
4.4.3	$x = -2$ or $x = -4$

PAPER I

2.2.1	$x^2+2x+\left(\frac{1}{2}(2)\right)^2+y^2-4y+\left(\frac{1}{2}(-4)\right)^2=5+1+4$ $(x+1)^2+(y-2)^2=10$ But $(x;y)\rightarrow(x-2;y+4)$ $\therefore(-1;2)\rightarrow(-1-2;2+4)$ $=(-3;6)$ $\therefore(x+3)^2+(y-6)^2=10$ **OR** $(x+2)^2+2(x+2)+(y-4)^2-4(y-4)-5=0$ $x^2+y^2+6x-12y+35=0$ $(x+3)^2+(y-6)^2=10$

2.2.2	Distance from origin to centre $=\sqrt{(-3-0)^2+(6-0)^2}=\sqrt{45}$ Since $\sqrt{45}>\sqrt{10}$, the origin lies outside the circle.

Question 3

3.1	AB : $y+3x-2=0$ $\therefore y=-3x+2$ $\therefore k=2$
3.2	AC $=\sqrt{(1-6)^2+(-1-4)^2}$ $=\sqrt{50}$ $=5\sqrt{2}$
3.3	$m_{AB}=\frac{-1-2}{1-0}=-3$ $m_{BC}=\frac{4-2}{6-0}=\frac{1}{3}$ $m_{AB}\times m_{BC}=-3\times\frac{1}{3}=-1$ $\therefore A\hat{B}C=90^\circ$.

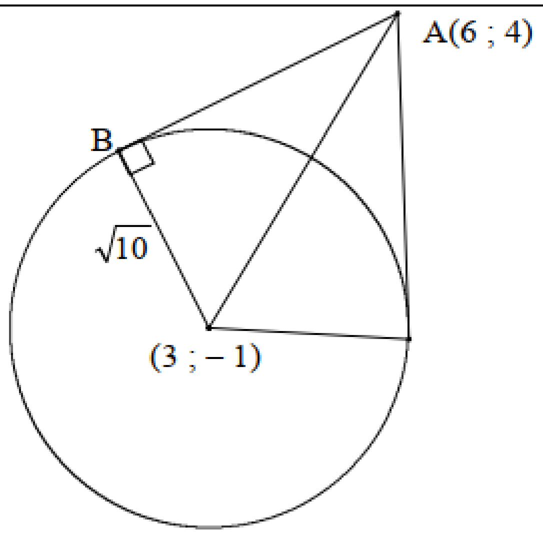
3.4	$\tan\beta = m_{AC} = 1$ $\beta = 45°$ $\tan\phi = m_{BC} = \frac{1}{3}$ $\phi = 18{,}43°$ $\theta = 45° - 18{,}43°$ [ext $\angle$of Δ] $= 26{,}57°$
3.5	$B\hat{D}C = 90^\circ$ altitude $\therefore$ BC is the diameter (line subtends $90^\circ \angle$) midpoint of BC : $\left(\frac{0+6}{2};\frac{2+4}{2}\right)$ $= (3;3)$ $(x-3)^2 + (y-3)^2 = r^2$ subst B$(0;2)$ $(0-3)^2 + (2-3)^2 = r^2$ $\therefore r^2 = 10$ $(x-3)^2 + (y-3)^2 = 10$
3.6	$M_{AC} = \left(\frac{1+6}{2};\frac{2+1}{2}\right)$ $= \left(\frac{7}{2};\frac{3}{2}\right)$ $\frac{7}{2} = \frac{x+0}{2}$ $\quad\quad$ $\frac{3}{2} = \frac{2+y}{2}$ $x = 7$ $\quad\quad$ $y = 1$ D(7 ; 1)

QUESTION 5

5.1.1	$x^2 + y^2 - 8x + 6y$ $= (2)^2 + (-9)^2 - 8(2) + 6(-9)$ $= 4 + 81 - 16 - 54$ $= 15$ Hence, the point lies on the circumference of the circle **OR** $x^2 + y^2 - 8x + 6y = 15$ $(x-4)^2 + (y+3)^2 = 15 + 16 + 9$ $(x-4)^2 + (y+3)^2 = 40$ $(x-4)^2 + (y+3)^2$ $= (2-4)^2 + (-9+3)^2$ $= 2^2 + 6^2$ $= 40$ $\therefore$ The point lies on the circumference of the circle.

5.1.2	$x^2 + y^2 - 8x + 6y = 15$ $(x-4)^2 + (y+3)^2 = 15 + 16 + 9$ $(x-4)^2 + (y+3)^2 = 40$ Circle centre $(4\,;-3)$ $m_{rad} = \frac{-3-(-9)}{4-2}$ $m_{rad} = 3$ $m_{\tan} = -\frac{1}{3}$ $y + 9 = -\frac{1}{3}(x-2)$ $y = -\frac{1}{3}x - \frac{25}{3}$

5.2



Radius $= \sqrt{10}$

Distance from A to centre of circle is

$= \sqrt{(6-3)^2 + (4+1)^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$

$AB^2 = 34 - 10$

$AB^2 = 24$

$AB = \sqrt{24}$

$AB = 2\sqrt{6}$

$AB = 4{,}90$

OR

$r^2 = 10$

$r = \sqrt{10}$

Radius $\perp$ tangent

By Pythagoras

$AB^2 = (6-3)^2 + (4+1)^2 - 10$

$= 24$

$AB = 4{,}90$

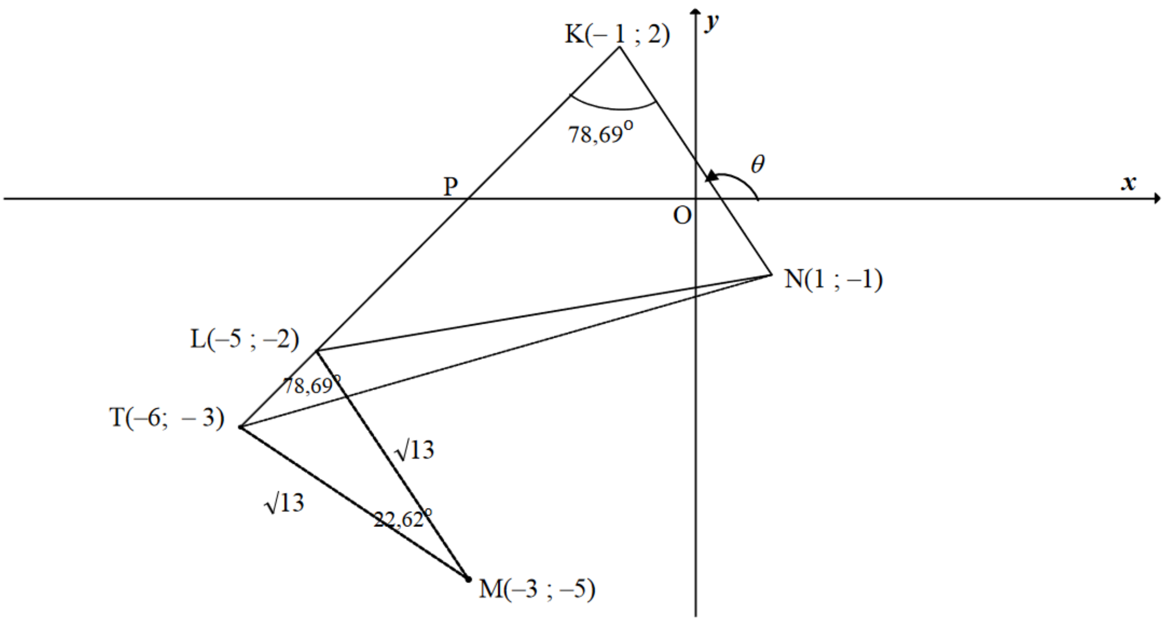
QUESTION 6

6.1	$x^2+y^2+8x+4y-38=0$ $x^2+8x+16+y^2+4y+4=16+4+38$ $(x+4)^2+(y+2)^2=58$ Centre is $(-4\,;-2)$ and the radius is $\sqrt{58}$
6.2	Centre of second circle is $(4\,;6)$ Distance between centres is $\sqrt{(4+4)^2+(6+2)}=\sqrt{128}=11{,}31$
6.3	Sum of radii $=\sqrt{58}+\sqrt{26}=12{,}71$ Distance between centres is $11{,}31$. sum of the radii > distance between the centres $\therefore$ the circles must overlap and hence the circles must intersect.
6.4	Equation of second circle: $(x-4)^2+(y-6)^2=26$ $x^2-8x+16+y^2-12y+36=26$ $x^2-8x+y^2-12y+26=0$ Let $(x\,;y)$ be either of the two points on intersection. Then $x^2+y^2+8x+4y-38=0$ and $x^2+y^2-8x-12y+26=0$ Subtract $16y+16x-64=0$ $y=-x+4$ Both points of intersection lie on this line. $\therefore\ y=-x+4$ is the equation of the common chord. **OR**

	Check that the line $y = -x + 4$ cuts the two circles at the same points: $(x-4)^2 + (-x-2)^2 = 26$ $x^2 - 8x + 16 + x^2 + 4x + 4 = 26$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ $x^2 + y^2 + 8x + 4y - 38 = 0$ $x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$ $x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$ $2x^2 - 4x - 6 = 0$ $x^2 - 2x - 3 = 0$ $x = 3$ or $x = -1$

PAPER J

QUESTION/*VRAAG* 3



3.1.1	$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \frac{2-(-1)}{-1-1}$ $= -\frac{3}{2}$
3.1.2	$\tan\theta = m_{KN} = -\frac{3}{2}$ $\theta = 180° - 56{,}31°$ $\theta = 123{,}69°$
3.2	Inclination KL = $123{,}69° - 78{,}69° = 45°$ [ext $\angle\Delta$] $\tan 45° = m_{KL} = 1$
3.3	$y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x-(-1))$ $y = x + 3$
3.4	$KN = \sqrt{(1+1)^2 + (-1-2)^2}$ $KN = \sqrt{13}$ or 3,61

3.5.1	$(x+3)^2+(y+5)^2=13$ ...(1) L is a point on KL $y=x+3$ ...(2) (2) in (1): $(x+3)^2+(x+3+5)^2=13$ $x^2+6x+9+x^2+16x+64=13$ $2x^2+22x+60=0$ $x^2+11x+30=0$ $(x+5)(x+6)=0$ $x=-5$ or $x=-6$ $y=-2$ or $y=-3$ L$(-5\,;-2)$ or $(-6\,;-3)$ **OR/OF** $(x+3)^2+(y+5)^2=13$ ...(1) L is a point on KL $y=x+3$ $\therefore x=y-3$ ...(2) (2) in (1): $(y-3+3)^2+(y+5)^2=13$ $y^2+y^2+10y+25=13$ $2y^2+10y+12=0$ $y^2+5y+6=0$ $(y+2)(y+3)=0$ $y=-2$ or $y=-3$ $x=-5$ or $x=-6$ L$(-5\,;-2)$ or $(-6\,;-3)$
3.5.2	Midpoint of KM: $(-2\,;-1{,}5)$ $\therefore \frac{x_L+1}{2}=-2$ and $\frac{y_L-1}{2}=-\frac{3}{2}$ $\therefore$ L$(-5\,;-2)$ **OR/*OF*** $m_{KN}=m_{LM}$ $\frac{y-(-5)}{x-(-3)}=-\frac{3}{2}$ $2(x+3+5)=-3(x+3)$ $2x+16=-3x-9$ $5x=-25$ $x=-5$ $\therefore$ L$(-5\,;-2)$

OR/*OF*

N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$

$\therefore$ L$(-1-4\,;2-4)$

$\therefore$ L$(-5\,;-2)$

OR/OF

N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$

$\therefore$ L$(-3-2\,;-5+3)$

$\therefore$ L$(-5\,;-2)$

OR/*OF*

N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$

$\therefore$ L$(-1-4\,;2-4)$

$\therefore$ L$(-5\,;-2)$

OR/OF

N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$

$\therefore$ L$(-3-2\,;-5+3)$

$\therefore$ L$(-5\,;-2)$

OR/*OF*

N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$

$\therefore$ L$(-1-4\,;2-4)$

$\therefore$ L$(-5\,;-2)$

OR/OF

N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$

$\therefore$ L$(-3-2\,;-5+3)$

$\therefore$ L$(-5\,;-2)$

OR/*OF*

N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$

$\therefore$ L$(-1-4\,;2-4)$

$\therefore$ L$(-5\,;-2)$

OR/OF

N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$

$\therefore$ L$(-3-2\,;-5+3)$

$\therefore$ L$(-5\,;-2)$

OR/*OF*

N→M: $(x\,;y)\rightarrow(x-4\,;y-4)$

$\therefore$ L$(-1-4\,;2-4)$

$\therefore$ L$(-5\,;-2)$

OR/OF

N→K: $(x\,;y)\rightarrow(x-2\,;y+3)$

$\therefore$ L$(-3-2\,;-5+3)$

$\therefore$ L$(-5\,;-2)$

3.6 T(–6 ; –3) (from Question 3.5.1)

$$\text{KT}=\sqrt{(-1-(-6))^2+(2-(-3))^2}$$
$$=\sqrt{50}$$

KN $=\sqrt{13}$ (CA from 3.4)

$$\text{Area of }\Delta\text{KTN}=\frac{1}{2}\text{KT.KN}\sin\text{L}\hat{\text{K}}\text{N}$$
$$=\frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69^\circ$$
$$=12{,}50\text{ square units}$$

OR/*OF*

In ΔKLM:

$$\frac{\text{TL}}{\sin 22{,}62°} = \frac{\sqrt{13}}{\sin 78{,}69°}$$

$$\text{TL} = 1{,}414..$$

$$\text{KL} = \sqrt{(-1-(-5))^2 + (2-(-2))^2}$$

$$= \sqrt{32}$$

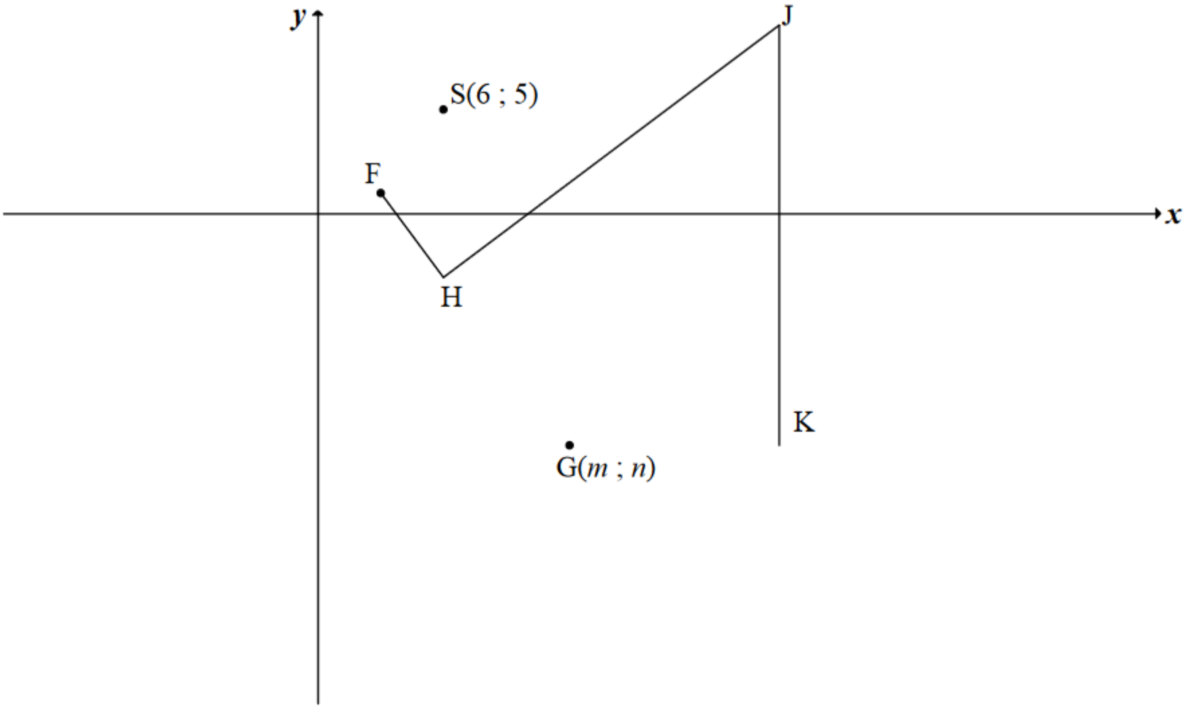
$\therefore$ KT = 7,0708…

$$\text{Area of } \Delta\text{KTN} = \frac{1}{2}\text{KT.KN}\sin \text{L}\hat{\text{K}}\text{N}$$

$$= \frac{1}{2}(7{,}0708).\sqrt{13}\sin 78{,}69°$$

$$= 12{,}50 \text{ square units}$$

QUESTION/*VRAAG* 4



4.1	F(3 ; 1)
4.2	$FS = \sqrt{(6-3)^2 + (5-1)^2}$ $FS = 5$
4.3	FH(FS) : HG = 1 : 2 $\therefore$ HG = 2 FH = 10
4.4	Tangents from common/same point / *Raaklyne vanaf gemeenskaplike of dieselfde punt*
4.5.1	$F\hat{H}J = 90°$ [tan ⊥ radius / *rkl ⊥ radius*] $FJ^2 = 20^2 + 5^2$ [Pyth theorem/*stelling*] $FJ = \sqrt{425}$ or $5\sqrt{17}$ or 20,62
4.5.2	$(x-m)^2 + (y-n)^2 = 100$

4.5.3	K(22; n) [radius $\perp$ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ F, H and G are collinear [HJ is a common tangent] *F, H en G is saamlynig [HJ is 'n gemeemskaplike raaklyn]* $FG^2 = (12-3)^2 + (n-1)^2$ $15^2 = 81 + (n-1)^2$ $(n-1)^2 = 144$ $n - 1 = \pm 12$ **OR/*OF*** $n^2 - 2n - 143 = 0$; $(n+11)(n-13) = 0$; $n = -11$ or $n \neq 13$ $n \neq 13$ or $n = -11$ $\therefore$ G(12; –11) **OR/*OF*** K(22; n) [radius $\perp$ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ Let J(22 ; y): $FJ^2 = (22-3)^2 + (y-1)^2$ $425 = 361 + y^2 - 2y + 1$ $0 = y^2 - 2y - 63$ $0 = (y-9)(y+7)$ $\therefore y = 9$ or/of $y \neq -7$ $\therefore n = 9 - 20 = -11$ $\therefore$ G(12; –11)

TRIGONOMETRY

PAPER A

QUESTION 5

5.1.1	$\sin(360°+x)$ $=\sin x$
5.1.2	$x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(-3)^2}$ $=-2$ $\tan x=\dfrac{-3}{-2}$ $=\dfrac{3}{2}$ **OR/*OF*** $x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(3)^2}$ $=2$ $\tan x=\dfrac{3}{2}$
5.1.3	$\cos(180°+x)$ $=-\cos x$

5.2	$\dfrac{\cos(90°+\theta)}{\sin(\theta-180°)+3\sin(-\theta)}$ $=\dfrac{-\sin\theta}{\sin(-(180°-\theta))-3\sin\theta}$ $=\dfrac{-\sin\theta}{-\sin\theta-3\sin\theta}$ $=\dfrac{-\sin\theta}{-4\sin\theta}$ $=\dfrac{1}{4}$

5.3	$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ $\cos x + 2\sin x = 0$ or $3\sin 2x - 1 = 0$ $\tan x = -\frac{1}{2}$; $\sin 2x = \frac{1}{3}$ ref $\angle = 26{,}565...^\circ$; ref $\angle = 19{,}471...^\circ$ $x = 153{,}43^\circ + k.180^\circ$; $k \in Z$; $x = 9{,}74^\circ + k.180^\circ$; $k \in Z$ **OR/*OF*** ; or $x = 153{,}43^\circ + k.360^\circ$; $k \in Z$; $x = 80{,}26^\circ + k.180^\circ$; $k \in Z$ or $x = 333{,}43^\circ + k.360^\circ$; $k \in Z$
5.4.1	LHS $= \cos(x+y).\cos(x-y)$ $= [\cos x.\cos y - \sin x.\sin y][\cos x.\cos y + \sin x.\sin y]$ $= \cos^2 x.\cos^2 y - \sin^2 x.\sin^2 y$ $= (1-\sin^2 x)(1-\sin^2 y) - \sin^2 x.\sin^2 y$ $= 1 + \sin^2 x.\sin^2 y - \sin^2 x - \sin^2 y - \sin^2 x.\sin^2 y$ $= 1 - \sin^2 x - \sin^2 y \quad =$ RHS

5.4.2	$1 - \sin^2 45^\circ - \sin^2 15^\circ$ $= \cos(45^\circ + 15^\circ).\cos(45^\circ - 15^\circ)$ $= \cos 60^\circ.\cos 30^\circ$ $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4}$ **OR/*OF***

$$1-\sin^2 45°-\sin^2 15°$$
$$=\sin^2 15°+\cos^2 15°-\sin^2 45°-\sin^2 15°$$
$$=\cos^2 15°-\left(\frac{\sqrt{2}}{2}\right)^2$$
$$=\cos^2 15°-\frac{1}{2}$$
$$=\frac{2\cos^2 15°-1}{2}$$
$$=\frac{\cos 30°}{2}$$
$$=\frac{\sqrt{3}}{2}\times\frac{1}{2}$$
$$=\frac{\sqrt{3}}{4}$$

OR

$$1-\sin^2 45°-\sin^2 15°$$
$$=\cos^2 45°-\sin^2(45°-30°)$$
$$=\left(\frac{1}{\sqrt{2}}\right)^2-(\sin 45°\cos 30°-\cos 45°\sin 30°)^2$$
$$=\frac{1}{2}-\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)^2$$
$$=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$$
$$=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$$
$$=\frac{1}{2}-\left(\frac{3}{8}-\frac{\sqrt{3}}{4}+\frac{1}{8}\right)$$
$$=\frac{\sqrt{3}}{4}$$

5.5.1	$16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 8\sin x.\cos x(2\cos^2 x - 1)$ $= 4\sin 2x(\cos 2x)$ $= 2\sin 4x$ **OR/*OF*** $16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 16\cos^2 x\left(\frac{1}{2}\sin 2x\right) - 8\left(\frac{1}{2}\sin 2x\right)$ $= 8\left(2\cos^2 x - 1\right)\left(\frac{1}{2}\sin 2x\right)$ $= 4\sin 2x.\cos 2x$ $= 2\sin 4x$
5.5.2	$16\sin x.\cos^3 x - 8\sin x.\cos x = 2\sin 4x$ Minimum at $x = 67{,}5°$

QUESTION 6

6.1	$180°$
6.2.1	$k=\sqrt{3}=1{,}73$
6.2.2	$\mathrm{B}\left(-120°;\sqrt{3}\right)$
6.3	Range of g: $y\in[-2;\ 2]$ Range of $2g(x)$: $y\in[-4;\ 4]$ **OR/*OF*** Range of g: $-2\le y\le 2$ Range of $2g(x)$: $-4\le y\ \le 4$
6.4	$x\in[-65°;\ -5°]$ **OR/*OF*** $-65°\le x\ \le -5°$
6.5	$\sin x.\cos x=p$ $4\sin x.\cos x=4p$ $2\sin 2x=4p$ $4p\ =\pm 2$ $\therefore\ p\ =\ -\frac{1}{2}$ or $\frac{1}{2}$

QUESTION 7

7.1	$AD^2 = AB^2 + BD^2$ $AD^2 = (\sqrt{5}p)^2 + (2p)^2$ $AD^2 = 9p^2$ $AD = 3p$
7.2	$\frac{CD}{\sin(135° - x)} = \frac{3p}{\sin x}$ $CD = \frac{3p\sin(135° - x)}{\sin x}$ $CD = \frac{3p(\sin 135° \cos x - \cos 135° \sin x)}{\sin x}$ $CD = \frac{3p(\sin 45° \cos x + \cos 45° \sin x)}{\sin x}$ $CD = \frac{3p\left(\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x\right)}{\sin x}$ $CD = \frac{3p\left(\frac{\sqrt{2}}{2}\right)(\cos x + \sin x)}{\sin x}$ $CD = \frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$
7.3	$\text{Area } \Delta ADC = \frac{1}{2}(AD)(CD)\sin A\hat{D}C$ $= \frac{1}{2}(3p)\left(\frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}\right)(\sin 45°)$ $= \frac{1}{2}(30)\left(\frac{30(\sin 110° + \cos 110°)}{\sqrt{2}\sin 110°}\right)\sin 45°$ $= 143{,}11 m^2$

PAPER B

QUESTION/VRAAG 5

5.1	$\tan(-x).\cos x.\sin(x-180°)-1$ $=-\tan x.\cos x.\sin(-(180°-x))-1$ $=\dfrac{-\sin x}{\cos x}.\cos x.(-\sin x)-1$ $=\sin^2 x-1$ $=-\cos^2 x$
5.2.1	$\cos 215°$ $=-\cos 35°$ $=-m$
5.2.2	$\sin 20°$ $=\cos 70°$ $=\cos 2(35°)$ $=2\cos^2 35°-1$ $=2m^2-1$ OR $=\sin(55°-35°)$ $=\sin 55°\cos 35°-\cos 55°\sin 35°$ $=m.m-\sqrt{1-m^2}.\sqrt{1-m^2}$ $=m^2-(1-m^2)$ $=2m^2-1$

5.3	$\cos 4x.\cos x + \sin 4x.\sin x = -0{,}7$ $\cos(4x - x) = -0{,}7$ $\text{ref}\angle = 45{,}57\ldots°$ $3x = 180° - 45{,}57\ldots° + k.360°$ or $3x = 180° + 45{,}57\ldots° + k.360°$ $3x = 134{,}43° + k.360°$ or $3x = 225{,}57° + k.360°$ $x = 44{,}81° + k.120°\ ;\ k \in Z$ $\quad x = 75{,}19° + k.120°\ ;\ k \in Z$
5.4	$\text{RHS} = \cos^2 x - \sin^2 x$ $\text{LHS} = \dfrac{\sin 4x.\cos 2x - 2\cos 4x.\sin x.\cos x}{\tan 2x}$ $= \dfrac{\sin 4x.\cos 2x - \cos 4x.\sin 2x}{\dfrac{\sin 2x}{\cos 2x}}$ $= \sin(4x - 2x)\left(\dfrac{\cos 2x}{\sin 2x}\right)$ $= \cos 2x$ $= \cos^2 x - \sin^2 x$ $\text{LHS} = \text{RHS}$
6.1	$1 - 2\sin^2 x = -\sin x$ $2\sin^2 x - \sin x - 1 = 0$ $(2\sin x + 1)(\sin x - 1) = 0$ $\sin x = -\dfrac{1}{2}$ or $\sin x = 1$ $\text{ref}\angle = 30°$ $\quad$ $\text{ref}\angle = 90°$ $x = 210° + k.360°$ $\quad$ $x = 90° + k.360°$ or $x = 330° + k.360°$ $x = -150°$ or $x = -30°$ or $x = 90°$

OR

$\cos 2x = -\sin x$
$\cos 2x = -\cos(90° - x)$

$2x = 180° - (90° - x) + k.360°$ or $2x = 180° + (90° - x) + k.360°$
$2x = 90° + x + k.360°$ or $2x = 270° - x + k.360°$
$x = 90° + k.360°$ $\quad$ $x = 90° + k.120°$

$x = -150°$ or $x = -30°$ or $x = 90°$

OR

$\cos 2x = -\sin x$
$\cos 2x = \cos(90° + x)$

$2x = 90° + x + k.360°$ or $2x = 360° - (90° + x) + k.360°$
$x = 90° + k.360°$ or $3x = 270° + k.360°$
$x = 90° + k.120°$

$x = -150°$ or $x = -30°$ or $x = 90°$

OR

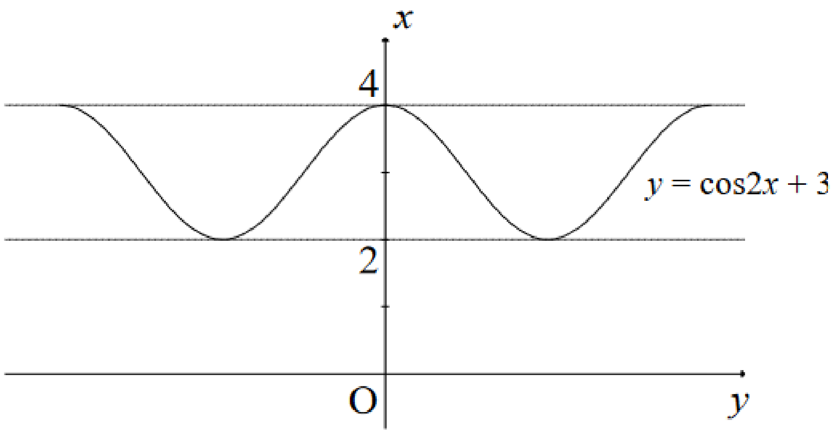
$\cos 2x = -\sin x$
$\sin(90° - 2x) = -\sin x$

$90° - 2x = 180 + x + k.360°$ or $90° - 2x = 360° - x + k.360°$

$x = -30° + k.120°$ $\quad$ $x = -270° + k.360°$

$x = -150°$ or $x = -30°$ or $x = 90°$

QUESTION 6

6.2.1	A(–150°; 0,5) B(–30°; 0,5) AB = –30° – (–150°) AB = 120°
6.2.2	$x \in (0°;\ 90°)$ or $x \in (90°;\ 180°)$ **OR** $0° < x < 90°$ or $90° < x < 180°$
6.2.3	$\cos 2x = k - 3$ $k - 3 < -1$ or $k - 3 > 1$ $k < 2$ or $k > 4$ **OR**  $k < 2$ or $k > 4$

QUESTION 7

7.1	In ΔBCE: $\frac{CE}{\sin\hat{B}} = \frac{BC}{\sin B\hat{E}C}$ $\frac{CE}{\sin 30^\circ} = \frac{BC}{\sin 2x}$ $CE = \frac{BC\sin 30^\circ}{\sin 2x}$ In ΔCDE: $\frac{DC}{CE} = \tan D\hat{E}C$ $DC = \frac{BC.\sin 30^\circ}{\sin 2x}(\tan x)$ $DC = \frac{BC}{4\sin x\cos x}\left(\frac{\sin x}{\cos x}\right)$ $DC = \frac{BC}{4\cos^2 x}$

7.2	$DC = \frac{BC}{4\cos^2 30^\circ}$ $= \frac{BC}{4\left(\frac{\sqrt{3}}{2}\right)^2}$ $= \frac{BC}{3}$ $\therefore BC = 3DC$	
	But AB = DC	[opp sides of rectangle/*teenoorst. sye v reghoek*]
	$\therefore BC = 3AB$	
	Area of rectangle	$= (AB)(BC)$ $= (AB)(3AB)$ $= 3AB^2$

PAPER C

QUESTION 5

5.1.1	$\text{OP}=\sqrt{(-7)^2+(4)^2}$ $=\sqrt{65}$
5.1.2(a)	$\tan\theta=\dfrac{4}{-7}$
5.1.2(b)	$\cos(\theta-180°) = -\cos\theta$ $= \dfrac{7}{\sqrt{65}}$

5.2	$\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.180°$; $k\in Z$ **OR/*OF*** $\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.360°$ or $x=251{,}57°+k.360°;\ k\in Z$

5.3.1	$\text{LHS} = \dfrac{\sin 3x}{1-\cos 3x} \times \dfrac{1+\cos 3x}{1+\cos 3x}$ $= \dfrac{(\sin 3x)(1+\cos 3x)}{(1-\cos 3x)(1+\cos 3x)}$ $= \dfrac{(\sin 3x)(1+\cos 3x)}{1-\cos^2 3x}$ $= \dfrac{(\sin 3x)(1+\cos 3x)}{\sin^2 3x}$ $= \dfrac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$
	OR/*OF* $\text{LHS} = \dfrac{\sin 3x}{1-\cos 3x} \times \dfrac{\sin 3x}{\sin 3x}$ $= \dfrac{\sin^2 3x}{\sin 3x(1-\cos 3x)}$ $= \dfrac{1-\cos^2 3x}{\sin 3x(1-\cos 3x)}$ $= \dfrac{(1-\cos 3x)(1+\cos 3x)}{\sin 3x(1-\cos 3x)}$ $= \dfrac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$
5.3.2	undefined when $\sin 3x = 0$ and $1-\cos 3x = 0$ $3x = 0°$ or $3x = 180°$ and $3x = 0°$ or $3x = 360°$ $x = 0°$ or $x = 60°$

QUESTION/*VRAAG* 6

6.1	$\dfrac{\sin 10°}{\cos 440°} + \tan(360° - \theta).\sin 2\theta$ $= \dfrac{\cos 80°}{\cos 80°} - \tan\theta(2\sin\theta\cos\theta)$ $= 1 - \dfrac{\sin\theta}{\cos\theta}(2\sin\theta\cos\theta)$ $= 1 - 2\sin^2\theta$ $= \cos 2\theta$
6.2.1	$\sin(60° + 2x) + \sin(60° - 2x) = k\cos 2x$ $(\sin 60°\cos 2x + \cos 60°\sin 2x) + (\sin 60°\cos 2x - \cos 60°\sin 2x) = k\cos 2x$ $2\sin 60°\cos 2x = k\cos 2x$ $2\left(\dfrac{\sqrt{3}}{2}\right)\cos 2x = k\cos 2x$ $\therefore k = \sqrt{3}$

6.2.2	$\tan 60°[\sin(60° + 2x) + \sin(60° - 2x)]$ $= \tan 60°[k\cos 2x]$ $= \sqrt{3}(\sqrt{3}\cos 2x)$ $= 3(2\cos^2 x - 1)$ $= 3\left(2\left(\sqrt{t}\right)^2 - 1\right)$ $= 6\left(\sqrt{t}\right)^2 - 3$ $= 6t - 3$

QUESTION 7

7.1	$A\left(0;\frac{1}{2}\right)$ $B\left(0;-\frac{1}{2}\right)$ $AB=\frac{1}{2}-\left(-\frac{1}{2}\right)$ $=1$ unit
7.2	Range of $f:\ y\in\left[-\frac{1}{2};\ \frac{1}{2}\right]$ Range of $3f(x)+2:\ \ y\in\left[\frac{1}{2};\ 3\frac{1}{2}\right]$ **OR/*OF*** $\frac{1}{2}\le y\le 3\frac{1}{2}$
7.3	$x=90°$
7.4.1	$x\in(30°;\ 90°)\cup(210°;240°]$ **OR/*OF*** $30°<x<90°$ or $210°<x\le 240°$
7.4.2	$x\in(-55°;\ 125°)$ **OR/*OF*** $-55°<x<125°$

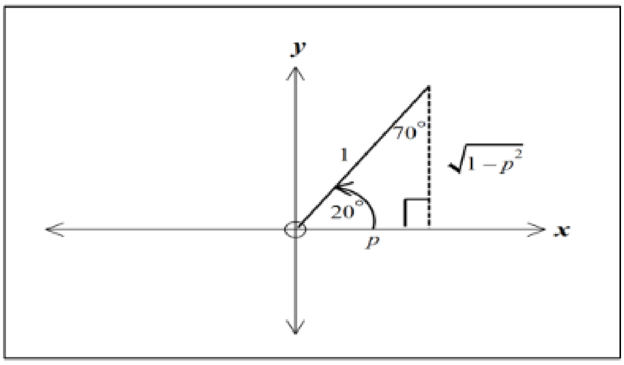
QUESTION 8

8.1	$\frac{0,5}{AB} = \sin 15°$ $AB = \frac{0,5}{\sin 15°}$ $AB = 1,93 \text{ m}$
8.2	$BE^2 = AB^2 + AE^2 - 2(AB)(AE)\cos B\hat{A}E$ $BE^2 = (1,93)^2 + (0,915)^2 - 2(1,93)(0,915)(\cos 120°)$ $BE = 2,52 \text{ m}$
8.3	$BF = FD = \frac{5}{7}(2,52) = 1,80 \text{ m}$ $\text{Area } \Delta BFD = \frac{1}{2}(BF)(FD)\sin B\hat{F}D$ $= \frac{1}{2}(1,8)(1,8)(\sin 75°)$ $= 1,56 \text{ m}^2$

PAPER D

QUESTION/*VRAAG* 5

5.1	$\dfrac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$ $=\dfrac{1-(-\sin\theta)(-\sin\theta)}{\cos\theta}$ $=\dfrac{1-\sin^2\theta}{\cos\theta}$ $=\dfrac{\cos^2\theta}{\cos\theta}$ $=\cos\theta$
5.2.1	$\cos 200°$ $=-\cos 20°$ $=-p$
5.2.2	$\sin(-70°)$ $=-\sin 70°$ $=-\cos 20°$ $=-p$ **OR/*OF*** $\sin(-70°)$ $=-\sin 70°$ $=-p$

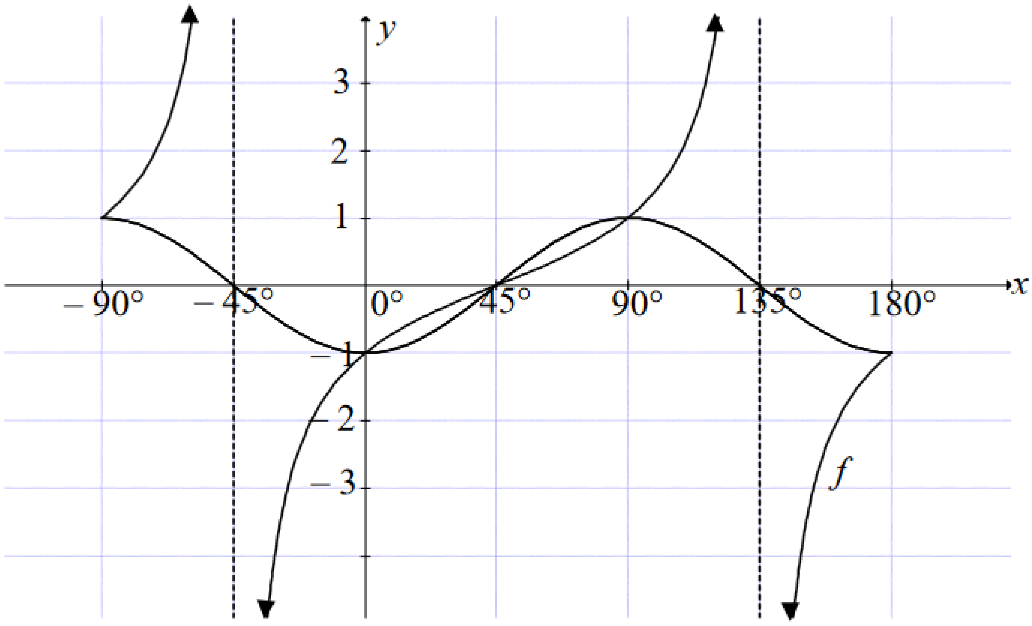


5.2.3	$\sin 10^\circ$ $\cos(2(10^\circ)) = 1 - 2\sin^2 10^\circ$ $2\sin^2 10^\circ = 1 - \cos 20^\circ$ $\sin 10^\circ = \sqrt{\dfrac{1-\cos 20^\circ}{2}}$ $\sin 10^\circ = \sqrt{\dfrac{1-p}{2}}$ **OR/*OF*** $\sin 10^\circ$ $\sin(30^\circ - 20^\circ)$ $= \sin 30^\circ \cos 20^\circ - \cos 30^\circ \sin 20^\circ$ $= \dfrac{1}{2}p - \dfrac{\sqrt{3}}{2}\sqrt{1-p^2} = \dfrac{p - \sqrt{3}\sqrt{1-p^2}}{2}$ **OR/*OF***
	$\sin 10^\circ$ $\sin(70^\circ - 60^\circ)$ $= \sin 70^\circ \cos 60^\circ - \cos 70^\circ \sin 60^\circ$ $= p.\dfrac{1}{2} - \sqrt{1-p^2} \times \dfrac{\sqrt{3}}{2} = \dfrac{p - \sqrt{3}\sqrt{1-p^2}}{2}$ **OR/*OF*** $\sin 10^\circ$ $= \cos 80^\circ$ $\cos(60^\circ + 20^\circ)$ $= \cos 60^\circ \cos 20^\circ - \sin 60^\circ \sin 20^\circ$ $= \dfrac{1}{2}p - \dfrac{\sqrt{3}}{2}.\sqrt{1-p^2}$

5.3	$\cos(A+55°)\cos(A+10°)+\sin(A+55°)\sin(A+10°)$ $=\cos[A+55°-(A+10°)]$ $=\cos 45°$ $=\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$
5.4.1	$\text{LHS}=\frac{\cos 2x+\sin 2x-\cos^2 x}{\sin x-2\cos x}$ $\quad$ $\text{RHS}=-\sin x$ $=\frac{\cos^2 x-\sin^2 x+2\sin x\cos x-\cos^2 x}{\sin x-2\cos x}$ $=\frac{-\sin^2 x+2\sin x\cos x}{\sin x-2\cos x}$ $=\frac{-\sin x(\sin x-2\cos x)}{\sin x-2\cos x}$ $=-\sin x$ $\therefore \text{LHS}=\text{RHS}$
5.4.2	$\frac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}$ $=\frac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin x(\sin x-2\cos x)}$ $=\frac{\cos 2x+\sin 2x-\cos^2 x}{(\sin x-2\cos x)}\times\frac{1}{-3\sin x}$ $=(-\sin x)\times\frac{1}{-3\sin x}$ $=\frac{1}{3}$

5.5.1	$3\tan 4x = -2\cos 4x$ $3\left(\dfrac{\sin 4x}{\cos 4x}\right) = -2\cos 4x$ $3\sin 4x + 2\cos^2 4x = 0$ $3\sin 4x + 2(1-\sin^2 4x) = 0$ $-2\sin^2 4x + 3\sin 4x + 2 = 0$ $2\sin^2 4x - 3\sin 4x - 2 = 0$ $(2\sin 4x + 1)(\sin 4x - 2) = 0$ $\sin 4x = -\dfrac{1}{2}$ or $\sin 4x \neq 2$
5.5.2	$\sin 4x = -\dfrac{1}{2}$ $ref.\ \angle = 30°$ $4x = 210° + k.360°$ or $4x = 330° + k.360°$ $x = 52{,}5° + k.90°\ ;\ k \in Z$ $x = 82{,}5° + k.90°\ ;\ k \in Z$

QUESTION/VRAAG 6

6.1	Period = $180°$
6.2	
6.3	$y \in [-1;1]$ **OR/*OF*** $-1 \leq y \leq 1$

6.4	$g(x) = -\cos 2x$ $g(x+45°) = -\cos 2(x+45°)$ $= -\cos(2x+90°)$ $= \sin 2x$
6.5.1	$x \in (-90°;-45°)$ **OR/*OF*** $-90° < x < -45°$
6.5.2	$2\cos 2x - 1 > 0$ $\cos 2x > \frac{1}{2}$ $-\cos 2x < -\frac{1}{2}$ $x \in (-30°;30°)$ **OR/*OF*** $-30° < x < 30°$

QUESTION 7

7.1.1	$\frac{AC}{20} = \cos 30°$ $AC = 20\cos 30°$ $AC = 10\sqrt{3} = 17{,}32$ units **OR/*OF*** $\frac{AC}{\sin 60°} = \frac{20}{\sin 90°}$ $\therefore AC = 20\sin 60 = 17{,}32$
7.1.2	$AB^2 = AC^2 + BC^2 - 2AC.BC\cos A\hat{C}B$ $AB^2 = \left(10\sqrt{3}\right)^2 + 8^2 - 2\left(10\sqrt{3}\right)(8)\cos 100°$ $AB = 20{,}30$ units
7.2	$\frac{\sin A\hat{D}B}{AB} = \frac{\sin A\hat{B}D}{AD}$ $\frac{\sin A\hat{D}B}{20{,}3} = \frac{\sin 73{,}4°}{20}$ $\sin A\hat{D}B = \frac{20{,}3\sin 73{,}4°}{20}$ $A\hat{D}B = 76{,}58°$

PAPER E

QUESTION 5

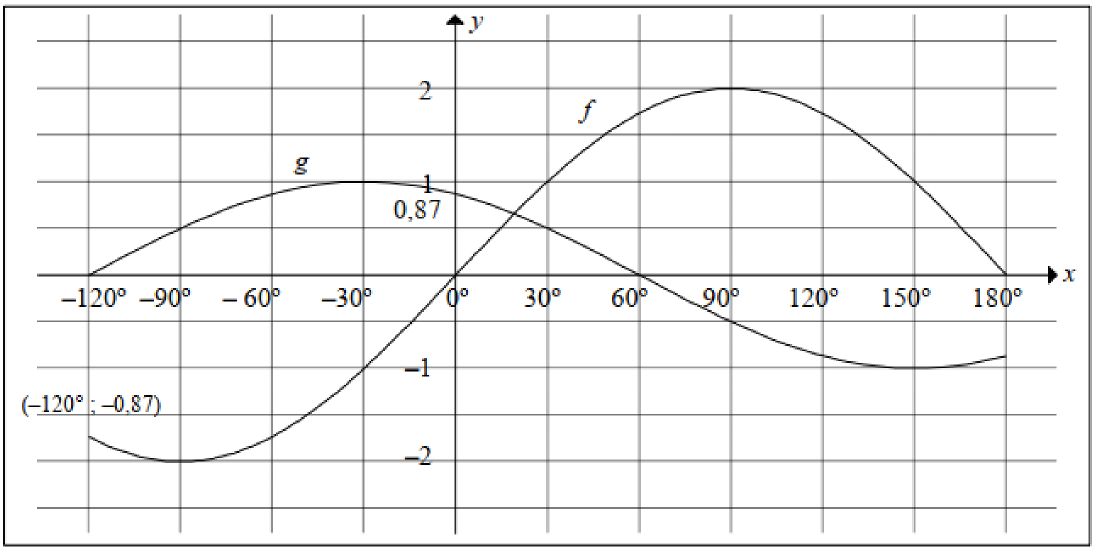
5.1.1	$\tan\theta = \dfrac{-\sqrt{3}}{1} = -\sqrt{3}$
5.1.2	$r^2 = (1)^2 + \left(-\sqrt{3}\right)^2$ Pyth $r^2 = 4$ $r = 2$ $\sin(-\theta) = -\sin\theta$ $= -\left(\dfrac{-\sqrt{3}}{2}\right)$ *or* $\dfrac{\sqrt{3}}{2}$
5.1.3	$\sin(\theta - 60°)$ $= \sin\theta\cos 60° - \cos\theta\sin 60°$ $= \left(-\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right) - \left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$ $= \dfrac{-\sqrt{3}}{4} - \dfrac{\sqrt{3}}{4} = \dfrac{-2\sqrt{3}}{4} = \dfrac{-\sqrt{3}}{2}$

5.2	$\dfrac{\tan(180° - \theta)\sin(90° + \theta)}{\cos 300° \sin(\theta - 360°)}$ $= \dfrac{-\tan\theta \,.\, \cos\theta}{\cos 60° \,.\, \sin\theta}$ $= \dfrac{-\dfrac{\sin\theta}{\cos\theta}.\cos\theta}{\dfrac{1}{2}.\sin\theta}$ $= -2$

5.3.1	$LHS = \dfrac{\cos 2x - 1}{\sin 2x}$ $= \dfrac{1 - 2\sin^2 x - 1}{2\sin x\cos x}$ $= -\dfrac{\sin x}{\cos x}$ $= -\tan x$ $= RHS$
5.3.2	$x = 90°\ ;\ 180°\ ;\ 270°$

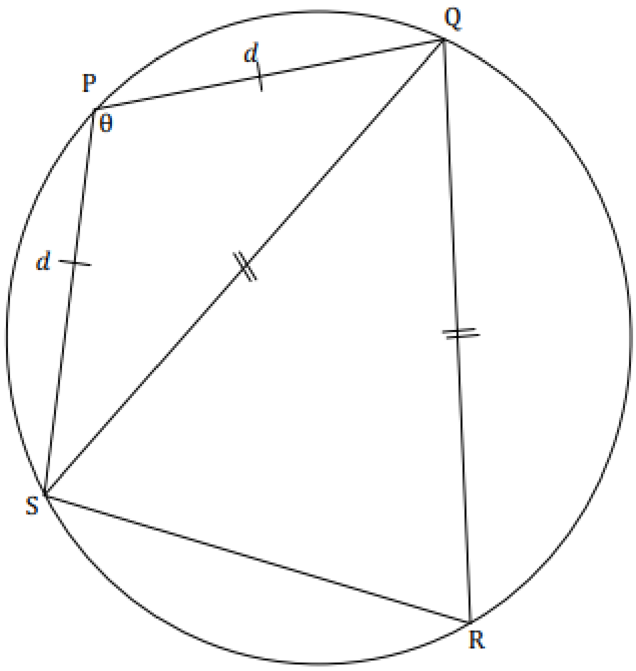
5.3.3	$-\tan 2x = \frac{1}{4}$ $\tan 2x = -\frac{1}{4}$ $2x = 165{,}96° + k.180°$ $x = 82{,}98° + k.90°\ ;\ k \in Z$

QUESTION 6

6.1	

6.2	Period $= 360°$
6.3.1	$-30° < x < 150°$
6.3.2	$x \in (0°\ ;60°)$
6.4	$y = \cos\left(x + 30° + 60°\right)$ $y = \cos\,(x + 90°)$ $y = -\sin x$

QUESTION 7



7.1	$QS^2 = d^2 + d^2 - 2d.d.\cos\theta$ $QS^2 = 2d^2 - 2d^2.\cos\theta$ $QS^2 = 2d^2(1-\cos\theta)$ $QS \ = d\sqrt{2(1-\cos\theta)}$

7.2	$\hat{R} = 180° - \theta$	opp. $\angle^s$cyclic quad suppl
	$= Q\hat{S}R$	equal sides, equal angles
	$S\hat{Q}R = 2\theta - 180°$	sum $\angle^s\Delta$
	$\Delta QRS = \frac{1}{2}.QS.QR\sin S\hat{Q}R$ $= \frac{1}{2}.d\sqrt{2(1-\cos\theta)}.d\sqrt{2(1-\cos\theta)}\sin(2\theta-180°)$ $= \frac{1}{2}d^2.2(1-\cos\theta)(-\sin 2\theta)$ $= -d^2(1-\cos\theta).\sin 2\theta$	

PAPER F

QUESTION / *VRAAG* 5

5.1	$1-4\sin^2 15°$ $=1-4\sin^2(45°-30°)$ $=1-4[\sin 45°\cos 30°-\cos 45°\sin 30°]^2$ $=1-4\left[\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}.\frac{1}{2}\right]^2$ $=1-4\left[\frac{\sqrt{6}-\sqrt{2}}{4}\right]^2$ $=1-4\left[\frac{6-4\sqrt{3}+2}{16}\right]$ $=1-4\left[\frac{8-4\sqrt{3}}{16}\right]$ $=1-\left[\frac{8-4\sqrt{3}}{4}\right]$ $=\sqrt{3}-1$
5.2	$\frac{\sqrt{3}\sin x.\sin^2 72°+\sin^2 198°.\sqrt{3}\cos(x-90°)}{\tan 120°.\sin x}$ $=\frac{\sqrt{3}\sin x.\sin^2(90°-18°)+\sin^2(180°+18°).\sqrt{3}\sin x}{\tan(180°-60°).\sin x}$ $=\frac{\sqrt{3}\sin x.\cos^2 18°+\sin^2 18°.\sqrt{3}\sin x}{-\tan 60°.\sin x}$ $=\frac{\sqrt{3}\sin x(\cos^2 18°+\sin^2 18°)}{-\sqrt{3}.\sin x}$ $=-1$

5.3

$$6\sin x.\cos x + 3\cos x - 4\sin^2 x - 2\sin x = 0$$
$$3\cos x(2\sin x + 1) - 2\sin x(2\sin x + 1) = 0$$
$$(2\sin x + 1)(3\cos x - 2\sin x) = 0$$
$$\sin x = -\frac{1}{2} \quad \textbf{OR/}\textbf{\textit{OF}} \quad 3\cos x = 2\sin x$$
$$\tan x = \frac{3}{2}$$

$\text{RA} = 30°$ $\qquad$ $\text{RA} = 56{,}31°$

$x = 210° + k.360°$ $\qquad$ $x = 56{,}31° + k.180°$

$x = 330° + k.360°$ $\qquad$ $x = 236{,}31° + k.180°; k \in Z$

5.4

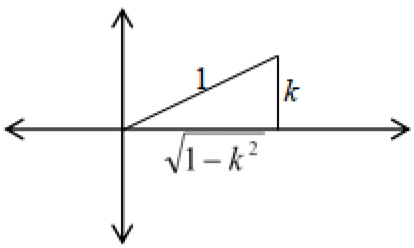
$$(1 - \tan \text{A})\left(\frac{\cos \text{A}}{\cos 2\text{A}}\right) = \frac{1}{\cos \text{A} + \sin \text{A}}$$
$$\text{LHS}/LK = (1 - \tan \text{A})\left(\frac{\cos \text{A}}{\cos 2\text{A}}\right)$$
$$= \left(1 - \frac{\sin \text{A}}{\cos \text{A}}\right)\left(\frac{\cos \text{A}}{\cos^2 \text{A} - \sin^2 \text{A}}\right)$$
$$= \left(\frac{\cos \text{A} - \sin \text{A}}{\cos \text{A}}\right)\left(\frac{\cos \text{A}}{(\cos \text{A} - \sin \text{A})(\cos \text{A} + \sin \text{A})}\right)$$
$$= \frac{1}{\cos \text{A} + \sin \text{A}}$$

$\text{LHS} / LK = \text{RHS} / RK$

5.5.1

$\cos 2\theta$

$= \sqrt{1 - k^2}$



OR/*OF*

$$\cos^2 2\theta = 1 - \sin^2 2\theta$$
$$= 1 - k^2$$
$$\cos 2\theta = \sqrt{1 - k^2}$$

5.5.2	$\frac{\sin 2\theta}{\tan\theta}$ $=\frac{2\sin\theta.\cos\theta}{\frac{\sin\theta}{\cos\theta}}$ $=2\sin\theta.\cos\theta.\frac{\cos\theta}{\sin\theta}$ $=2\cos^2\theta$ But/*maar* $\cos 2\theta=\sqrt{1-k^2}$ $2\cos^2\theta-1=\sqrt{1-k^2}$ $2\cos^2\theta=\sqrt{1-k^2}+1$ $\frac{\sin 2\theta}{\tan\theta}=\sqrt{1-k^2}+1$

QUESTION / *VRAAG* 6

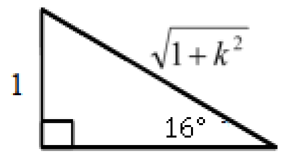
6.1	$a=-1$ $d=2$
6.2	$D\left(-150°;\frac{1}{2}\right)$
6.3.1	$-90°< x <90°$ **OR / *OF*** $x\in(-90°;90°)$
6.3.2	$-135°< x < -45°$ **OR / *OF*** $x\in(-135°;-45°)$

QUESTION 7

7.1	In Δ KLM $\frac{KM}{LM} = \tan x$ $\frac{KM}{r} = \tan x$ $KM = r\tan x$ In Δ KMN $\frac{MN}{KM} = \tan x$ $\frac{2r}{KM} = \tan x$ **OR/*OF*** $KM = \frac{2r}{\tan x}$ $\frac{2r}{r\tan x} = \tan x$ $r\tan x = \frac{2r}{\tan x}$ $2 = \tan^2 x$ $\sqrt{2} = \tan x$ $x = 54{,}74°$
7.2	$LN^2 = LM^2 + MN^2 - 2LM.MN\cos M$ $LN^2 = (5)^2 + (10)^2 - 2(5).(10)\cos 110°$ $LN^2 = 159{,}20$ $LN = \sqrt{159{,}20}$ $LN = 12{,}62\,m$

PAPER G

QUESTION / *VRAAG* 5

5.1.1	$x^2 = \left(\sqrt{1+k^2}\right)^2 - (1)^2$ (Pythagoras) $x^2 = k^2$ $x = k$ $\tan 16^\circ = \dfrac{1}{k}$ 
5.1.2	$\cos 32^\circ$ $= \cos 2(16^\circ)$ $= 2\cos^2 16^\circ - 1$ $= 2\left(\dfrac{k}{\sqrt{1+k^2}}\right)^2 - 1$

5.2	$\dfrac{\cos(90^\circ + x)\sin(x - 180^\circ) - \cos^2(180^\circ - x)}{\cos(-2x)}$ $= \dfrac{(-\sin x)(-\sin x) - \cos^2 x}{\cos 2x}$ $= \dfrac{\sin^2 x - \cos^2 x}{\cos^2 x - \sin^2 x}$ **OR / *OF*** $\dfrac{-\cos 2x}{\cos 2x}$ $= \dfrac{-(\cos^2 x - \sin^2 x)}{\cos^2 x - \sin^2 x}$ $= -1$ $= -1$

5.3	$\cos 75^\circ.\cos 45^\circ - \cos 15^\circ.\cos 45^\circ$ $= \cos 75^\circ.\cos 45^\circ - \sin 75^\circ.\sin 45^\circ$ $= \cos(75^\circ + 45^\circ)$ $= \cos 120^\circ$ $= -\cos 60^\circ$ $= -\dfrac{1}{2}$

5.4.1	$\tan\theta\left(\sin 2\theta+\dfrac{3\cos^2\theta}{\sin\theta}\right)$ $=\dfrac{\sin\theta}{\cos\theta}\left(2\sin\theta\cos\theta+\dfrac{3\cos^2\theta}{\sin\theta}\right)$ $=2\sin^2\theta+3\cos\theta$ $=2(1-\cos^2\theta)+3\cos\theta$ $=-2\cos^2\theta+3\cos\theta+2$
5.4.2	$-2\cos^2\theta+3\cos\theta+2=0$ $2\cos^2\theta-3\cos\theta-2=0$ $(2\cos\theta+1)(\cos\theta-2)=0$ $\cos\theta=-\dfrac{1}{2}$ **or / *of*** $\cos\theta=2$ no solution / *geen oplossing* ref / *verwy* $\angle=60^\circ$ $\theta=\pm120^\circ+k360^\circ;\ k\in Z$ **OR / *OF*** $\theta=120^\circ+k360^\circ;k\in Z$ $\theta=240^\circ+k360^\circ;k\in Z$

5.5	$\cos(a+b)=-\dfrac{\sqrt{2}}{2}$ ref $\angle$ / *verw* $\angle=45^\circ$ $a+b=180^\circ-45^\circ$ $a+b=135^\circ$ .... (1) $\cos(a-2b)=\dfrac{1}{2}$ ref $\angle$ / *verw* $\angle=60^\circ$ $a-2b=60^\circ$ .... (2) $3b=75^\circ$ (1)$-$(2) $b=25^\circ$ $a=110^\circ$

QUESTION 6

6.1	$x = -45°$ $x = 135°$

6.2	$h(x) = \tan(45° - x)$ $h(x) = -\tan(x - 45°) = -f(x)$ h is the reflection of f about the x-axis **OR** h is the reflection of f about the line $y = 0$

6.3	$y = 3\sin 2x$

QUESTION 7

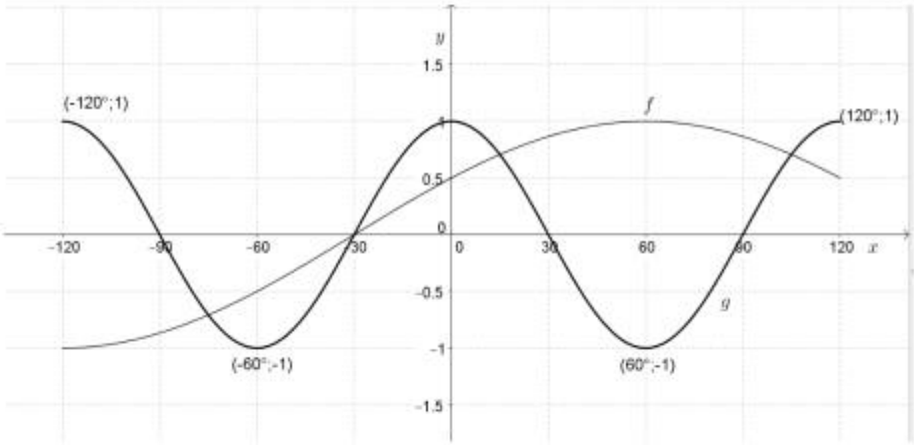
7.1	$M\hat{A}G = 2\theta$
7.2	$\frac{MG}{\sin 2\theta} = \frac{k}{\sin 90°}$ $MG = \frac{k\sin 2\theta}{1}$

7.3	$\frac{MC}{\sin 150°} = \frac{MG}{\sin\theta}$ $\frac{MC}{½} = \frac{MG}{\sin\theta}$ $MC = \frac{½MG}{\sin\theta}$ ……… (1) Subst. (2) in (1) $MC = \frac{k\sin 2\theta}{2\sin\theta}$ $MC = \frac{k2\sin\theta.\cos\theta}{2\sin\theta}$ $MC = k\cos\theta$

7.4	$\Delta MGC = \frac{1}{2} MG.CG\ sin150°$ $\Delta MGC = \frac{1}{2} ksin2\theta.8\ \frac{1}{2}$ $\Delta MGC = 2ksin2\theta$ **OR** $\Delta MGC = \frac{1}{2} MC.CG\ sin\theta$ $\Delta MGC = \frac{1}{2} kcos\theta.8\ sin\theta$ $\Delta MGC = 4kcos\theta sin\theta$ $\Delta MGC = 2k(2cos\theta sin\theta)$ $\Delta MGC = 2ksin2\theta$

PAPER H

QUESTION 6

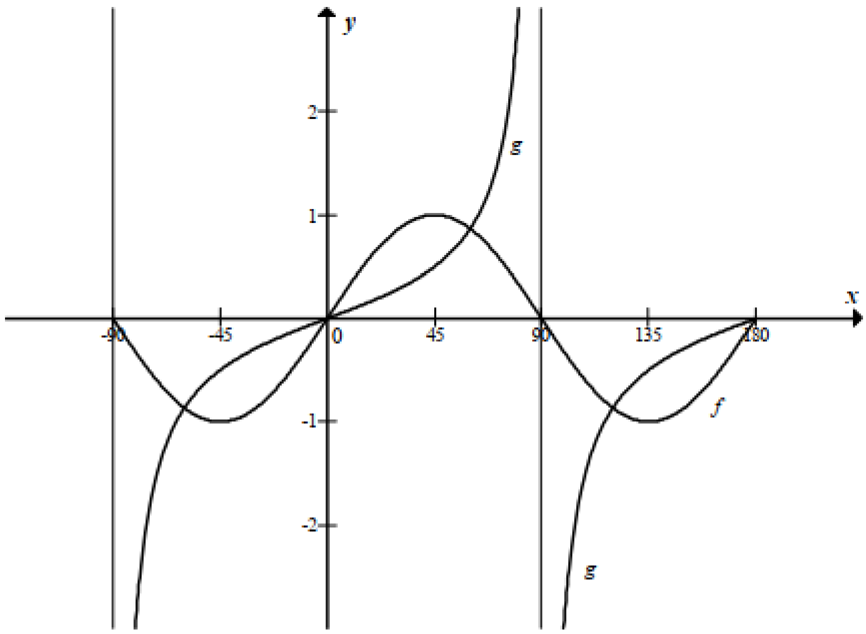
6.1	$f(x) = \sin(x + a)$
	

6.1	$a = 30°$
6.2	Sketch
6.3	$f(x) = g(x)$ $\sin(x + 30°) = \cos(3x)$ $\sin(x + 30°) = \sin(90° - 3x)$ $x + 30° = 90° - 3x \quad + 360°n \quad n \in Z$ $4x = 60° + 360°n$ $x = 15° + 90°n$ or $x + 30° = 180° - (90° - 3x) + 360°n$ $x + 30° = 90° + 3x + 360°n$ $-2x = 60° + 360°n$ $x = -30° - 180°n$
6.4	$15° < x < 105°$ or $x \in (15°; 105°)$
6. 5	$g(x) = \cos(3x)$ $k(x) = \cos(60° - 3x)$ $k(x) = \cos(3x - 60°)$ $k(x) = cos3(x - 20°)$ $\therefore translated\ 20°\ to\ the\ right.$

QUESTION 7

7.1	$\frac{7}{PB} = \sin 18°$ $PB = \frac{7}{\sin 18°}$ $PB = 22{,}65$ m (22,65247584...)
7.2	$\frac{18}{PA} = \cos 23°$ $PA = \frac{18}{\cos 23°}$ $PA = 19{,}55$ m (19,55448679..)
7.3	$AB^2 = (22{,}65)^2 + (19{,}55)^2 - 2(22{,}65)(19{,}55).\cos 42°$ $= 237{,}0847954...$ $AB = 15{,}40$ m (15,3975581...)

QUESTION 8

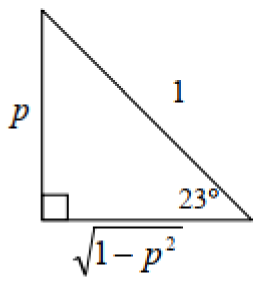
8.1	

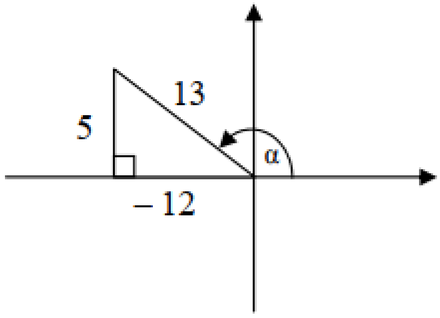
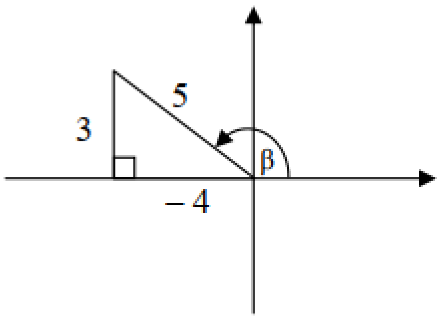
8.2	$\sin 2x = \frac{1}{2}\tan x$ $2\sin x.\cos x = \frac{\sin x}{2\cos x}$ $4\sin x.\cos^2 x - \sin x = 0$ $\sin x(4\cos^2 x - 1) = 0$ $\sin x = 0$ or $\cos^2 x = \frac{1}{4}$ $x = 0°$ or $180°$ $\cos x = \pm\frac{1}{2}$ $x = 60°\ ;\ -60°$ or $120°$

8.3	$\{x \mid -60° < x < 0°\} \cup \{x \mid 60° < x < 90°\} \cup \{x \mid 120° < x < 180°\}$ **OR** $x \in (-60°\ ;\ 0°) \cup (60°\ ;\ 90°) \cup (120°\ ;\ 180°)$ **OR** $-60° < x < 0°$ or $60° < x < 90°$ or $120° < x < 180°$

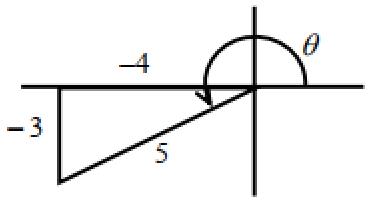
PAPER I

QUESTION 6

6.1.1	$\cos 113°$ $= \cos(90° + 23°)$ $= -\sin 23°$ $= -p$
6.1.2	$\cos 23°$ $= \sqrt{1-p^2}$  **OR** $\cos^2 23° + \sin^2 23° = 1$ $\cos^2 23° = 1 - p^2$ $\cos 23° = \sqrt{1-p^2}$

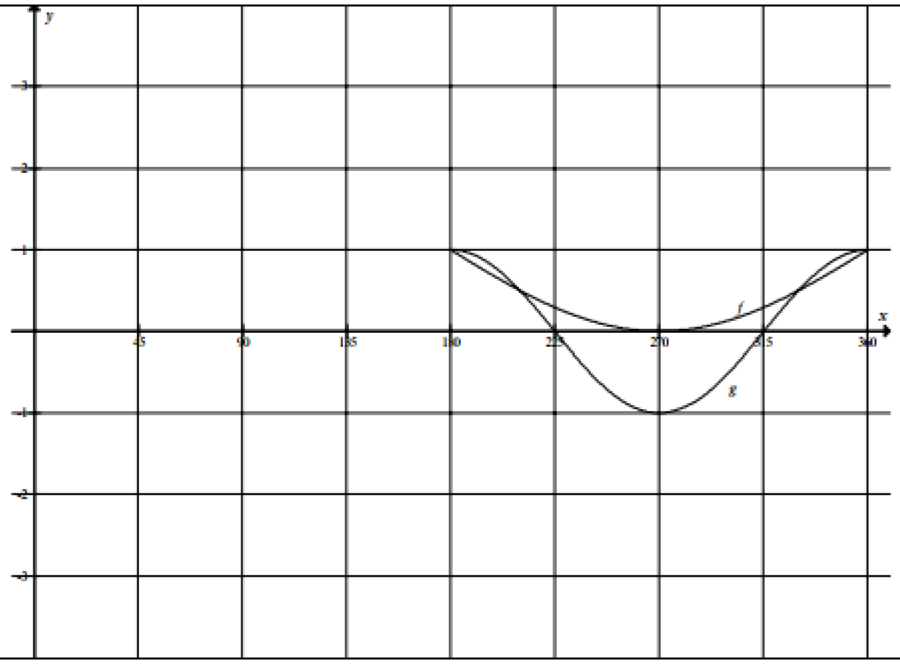
6.1.3	$\sin 46°$ $= 2\sin 23°.\cos 23°$ $= 2p\sqrt{1-p^2}$
6.2.1	$\sin\alpha = \frac{5}{13}$ $y_\alpha = 5 \quad r_\alpha = 13$ $x_\alpha = -12$ $\cos\alpha = -\frac{12}{13}$ 
6.2.2	$\tan\beta = -\frac{3}{4}$ $y_\beta = 3 \quad x_\beta = -4$ $r = 5$  $\cos(\alpha+\beta)$ $= \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right)$ $= \frac{48-15}{65}$ $= \frac{33}{65}$
6.3	$\frac{1}{2}\cos x = 0{,}435$ $\cos x = 0{,}87$ $x = 29{,}54°$ or $x = 330{,}46°$

QUESTION 9

9.1.1	$\sin\theta = -\frac{3}{5}$ and $\cos\theta = -\frac{4}{5}$ $\sin\theta + \cos\theta = -\frac{7}{5}$ 
9.1.2	$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ $= \frac{2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)}{\frac{16}{25} - \frac{9}{25}}$ $= \frac{24}{7}$
9.2.1	$\frac{\cos(360° - x).\tan^2 x}{\sin(x - 180°).\cos(90° + x)}$ $= \frac{(\cos x)(\tan^2 x)}{(-\sin x)(-\sin x)}$ $= (\cos x)\left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{1}{\sin^2 x}\right)$ $= \frac{1}{\cos x}$
9.2.2	$x = 30°$ $\frac{1}{\cos 30°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

QUESTION 11

11.1	$1 + \sin x = \cos 2x$ $1 + \sin x = 1 - 2\sin^2 x$ $\sin x + 2\sin^2 x = 0$ $\sin x(1 + 2\sin x) = 0$ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ $x = k.180$ or $x = -30° + k.360$ $k \in Z$ $x = 210° + k.360$ $x \in \{180°\,; 210; 330°\,; 360°\}$

11.2	
11.3	$180° \leq x \leq 210°$ or $330° \leq x \leq 360°$

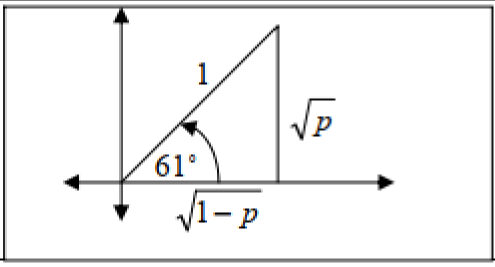
PAPER J

QUESTION 10

10.1.1	$\cos 28° = \sqrt{1-\sin^2 28°}$ $= \sqrt{1-a^2}$
10.1.2	$\cos 64°$ $= \cos 2(32°)$ $= 2\cos^2 32° - 1$ $= 2b^2 - 1$
10.1.3	$\sin 4°$ $= \sin(32° - 28°)$ $= \sin 32° \cos 28° - \cos 32° \sin 28°$ $= \sqrt{1-b^2}.\sqrt{1-a^2} - ab$

10.2	$b\sqrt{1-a^2}-a\sqrt{1-b^2}$ $=\cos 32°.\sqrt{1-\sin^2 28°}-\sin 28°\sqrt{1-\cos^2 32°}$ $=\cos 32°.\cos 28°-\sin 28°.\sin 32°$ $=\cos(32°+28°)$ $=\cos 60°$ $=\frac{1}{2}$

QUESTION 11

11.1.1	$\sin 61° = \sqrt{p}$ $\sin 241° = \sin(180° + 61°)$ $= -\sin 61°$ $= -\sqrt{p}$ 
11.1.2	$\cos 61° = \sqrt{1-\sin^2 61°}$ $=\sqrt{1-p}$
11.1.3	$\cos 122° = \cos 2(61°)$ $=2\cos^2 61°-1$ $=2\left(\sqrt{1-p}\right)^2-1$ $=2(1-p)-1$ $=2-2p-1$ $=1-2p$
11.1.4	$\cos 73°\cos 15° + \sin 73°.\sin 15°$ $=\cos(73° - 15°)$ $=\cos 58° = (\cos 180° - 122°)$ $=-(\cos 122°)$ $=-(1-2p)$ $=2p-1$

11.2.1	$\text{LHS} = \dfrac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$ $= \dfrac{\cos^2 x+2\cos x\sin x+\sin^2 x-(\cos^2 x-2\sin x\cos x+\sin^2 x)}{(\cos x-\sin x)(\cos x+\sin x)}$ $= \dfrac{4\cos x\sin x}{\cos^2 x-\sin^2 x}$ $= \dfrac{2\sin 2x}{\cos 2x}$ $= 2\tan x$ $= \text{RHS}$
11.2.2	$\cos x=\sin x$ or $\cos x=-\sin x$ $x=45°$ $\quad x=135°$
11.3.1	$\sin x=\cos 2x-1$ $\sin x=1-2\sin^2 x-1$ $\sin x=-2\sin^2 x$ $2\sin^2 x+\sin x=0$

11.3.2	$\sin x=\cos 2x-1$ $2\sin^2 x+\sin x=0$ $\sin x(2\sin x+1)=0$ $\sin x=0$ or $\sin x=-\dfrac{1}{2}$ $\therefore x=0°+180°k;\ k\in Z$ or $x=\{210° \text{ or } 330°\}+360°k;\ k\in Z$ **OR** $x=n.180°$ $x=n.360°-30°$ $x=(2n+1).180°+30°, n\in Z$

11.4	$\tan 1°\times\tan 2°\times\tan 3°\times\tan 4°\times\ldots\ldots\times\tan 87°\times\tan 88°\times\tan 89°$ $=\left(\dfrac{\sin 1°}{\cos 1°}\right)\left(\dfrac{\sin 2°}{\cos 2°}\right)\cdots\left(\dfrac{\sin 45°}{\cos 45°}\right)\cdots\left(\dfrac{\sin 88°}{\cos 88°}\right)\left(\dfrac{\sin 89°}{\cos 89°}\right)$ $=\left(\dfrac{\sin 1°}{\cos 1°}\right)\left(\dfrac{\sin 2°}{\cos 2°}\right)\cdots\left(\dfrac{\sin 45°}{\cos 45°}\right)\cdots\left(\dfrac{\sin(90°-2°)}{\cos(90°-2°)}\right)\left(\dfrac{\sin(90°-1°)}{\cos(90°-1°)}\right)$ $=\left(\dfrac{\sin 1°}{\cos 1°}\right)\left(\dfrac{\sin 2°}{\cos 2°}\right)\cdots\left(\dfrac{\sin 45°}{\cos 45°}\right)\cdots\left(\dfrac{\cos 2°}{\sin 2°}\right)\left(\dfrac{\cos 1°}{\sin 1°}\right)$ $=\tan 45°$ $=1$

EUCLID'S GEOMETRY

PAPER A

A1

QUESTION 7

7.1.1 equal to twice the angle subtended by the same chord at the circle.

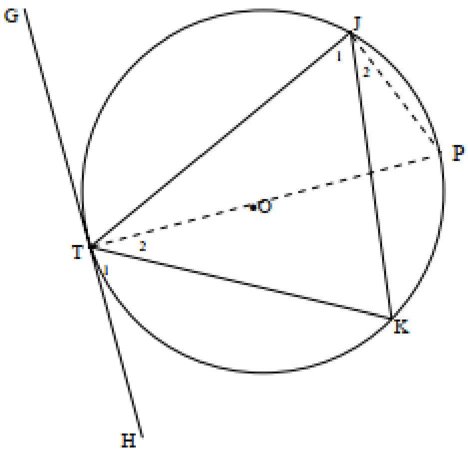
7.1.2 equal to the angle subtended by the same chord in the alternate segment.

7.1.3 supplementary.

A2

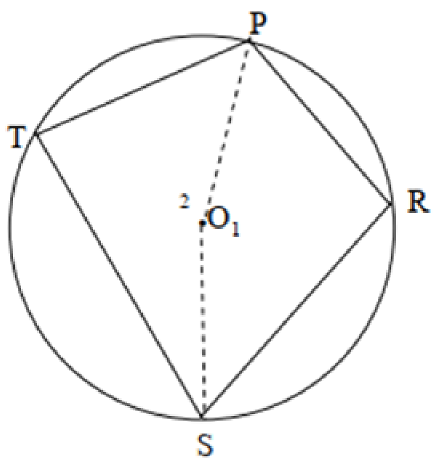
QUESTION 8

8.1	Draw diameter TP. Join P to J. $\hat{T}_1 + \hat{T}_2 = 90°$ (tan ⊥ diameter) $\hat{J}_1 + \hat{J}_2 = 90°$ ($\angle$ in semi-circle) $\hat{J}_2 = \hat{T}_2$ ($\angle$ in same seg) $T\hat{J}K = \hat{T}_1$



A3

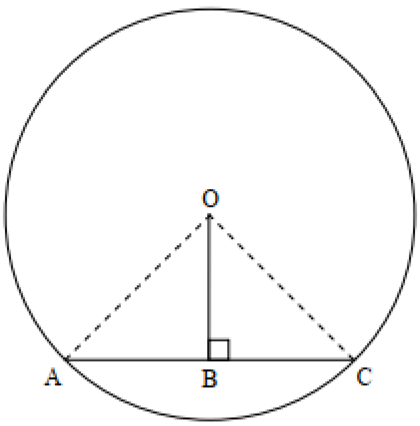
QUESTION 9



9.1	Join PO and OS
	Let $\hat{O}_1 = 2x$
	$\hat{T} = x$ (∠ at circ centre = 2 ∠ at circumference)
	$\hat{O}_2 = 360° - 2x$ (∠s round a point)
	$\hat{R} = 180° - x$ (∠ at circ centre = 2 ∠ at circumference)
	$\hat{T} + \hat{R} = x + 180° - x$
	$= 180°$

A4

QUESTION/*VRAAG* 11



Construct radii OA and OC.

In ΔOAB and ΔOCB

i. OB is common

ii. OA = OC (radii)

iii. $O\hat{B}A = O\hat{B}C = 90°$ (given)

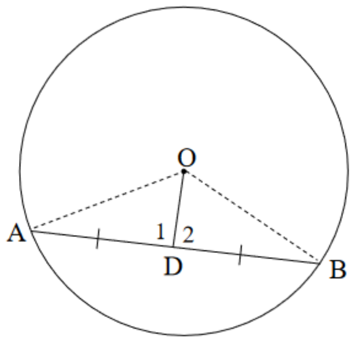
ΔOAB ≡ ΔOCB (90°HS)

AB = BC (≡ Δs)

A5

QUESTION/*VRAAG* 9

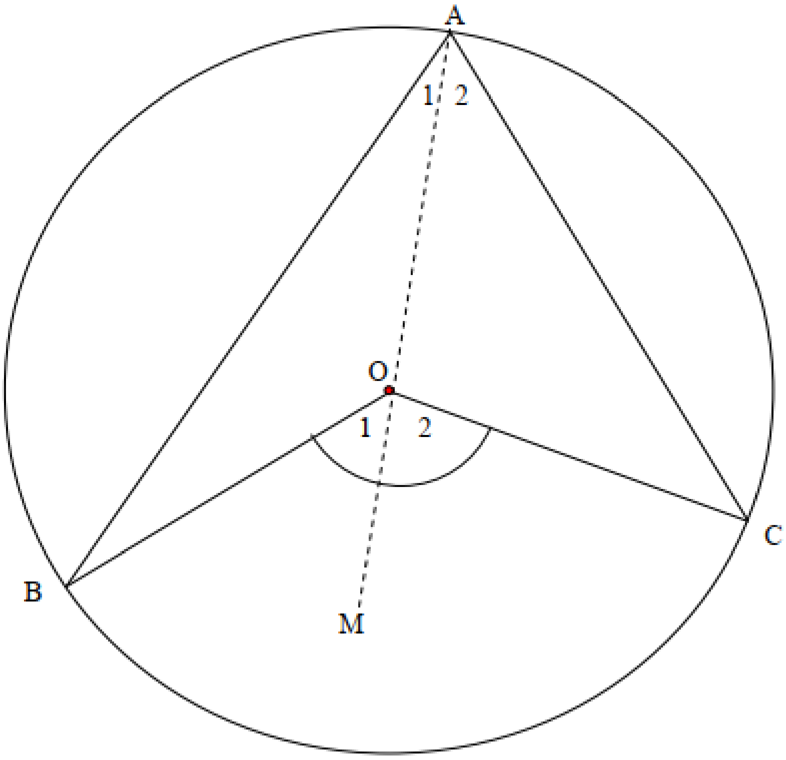
9.1



9.1.1	Construction:	
	Draw OA and OB	
	In ΔADO and ΔBDO	
	$OA = OB$	[radii/*radiusse*]
	$OD = OD$	[common side/*gemeenskaplike sy*]
	$AD = DB$	[given/*gegee*]
	$\therefore \Delta ADO \equiv \Delta BDO$	[S;S;S]
	ADB is a straight line	
	$\therefore \hat{D}_1 = \hat{D}_2$	$\Delta ADO \equiv \Delta BDO$
	$\therefore OD \perp AB$	[$\angle$s on a str line/$\angle$*e op 'n reguitlyn*]
	OR/*OF*	
	Construction:	
	Draw OA and OB	
	In ΔADO and ΔBDO	
	$AD = DB$	[given/*gegee*]
	$\hat{A} = \hat{B}$	[$\angle$s opp; $\angle$s sides /$\angle$*e teenoor gelyke sye*]
	$OA = OB$	[radii/*radiusse*]
	$\therefore \Delta ADO \equiv \Delta BDO$	[S;$\angle$;S]
	ADB is a straight line	
	$\therefore \hat{D}_1 = \hat{D}_2$	$\Delta ADO \equiv \Delta BDO$
	$\therefore OD \perp AB$	[$\angle$s on a str line/$\angle$*e op 'n reguitlyn*]

A6

QUESTION/*VRAAG* 10

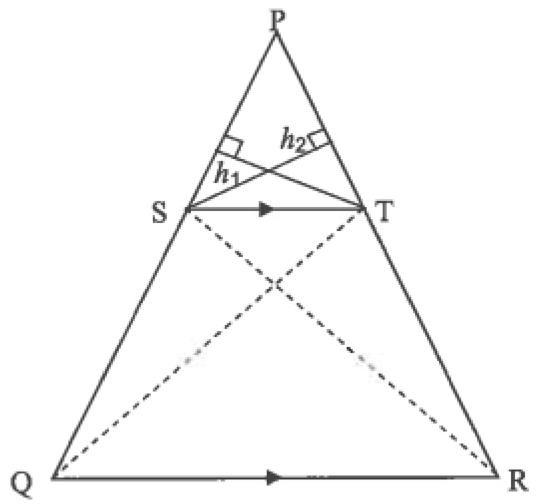


10.1	Construction: AO is drawn and produced to M $\hat{O}_1 = \hat{A}_1 + \hat{B}$ [ext ∠ of Δ/*buite ∠ van Δ*] But $\hat{A}_1 = \hat{B}$ [∠s opp = radii/*∠e teenoor =radii*] $\therefore \hat{O}_1 = 2\hat{A}_1$ Similarly/*Netso*: $\hat{O}_2 = 2\hat{A}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$ $= 2(\hat{A}_1 + \hat{A}_2)$ $B\hat{O}C = 2B\hat{A}C$

A7

QUESTION/*VRAAG* 10

10.1

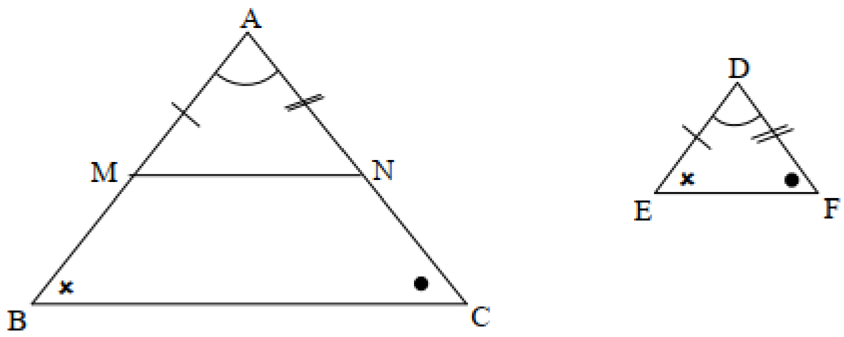


10.1	Constr : Join S to R and T to Q and draw h_1 from S $\perp$ PT and h_2 from T $\perp$ PS/ *Verbind SR en TQ en trek h_1 van S $\perp$ PT en h_2 van T $\perp$ PS*] Proof : $\frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\frac{1}{2}PS \times h_2}{\frac{1}{2}SQ \times h_2} = \frac{PS}{SQ}$ equal altitudes $\frac{\text{area } \Delta PST}{\text{area } \Delta STR} = \frac{\frac{1}{2}PT \times h_1}{\frac{1}{2}TR \times h_1} = \frac{PT}{TR}$ equal altitudes area ΔPST = area ΔPST [common] But area ΔQST = area ΔSTR [same base, height; ST $\parallel$ QR] $\therefore \frac{\text{area } \Delta PST}{\text{area } \Delta QST} = \frac{\text{area } \Delta PST}{\text{area } \Delta STR}$ $\therefore \frac{PS}{SQ} = \frac{PT}{TR}$

A8

QUESTION/*VRAAG* 10

10.1



10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN. *Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat AM = DE en AN = DF. Verbind MN.* Proof: In Δ AMN and Δ DEF AM = DE [Constr] AN = DF [Constr] $\hat{A} = \hat{D}$ [Given] $\therefore\ \Delta AMN \equiv \Delta DEF\,(SAS)$ $\therefore A\hat{M}N = \hat{E} = \hat{B}$ MN ‖ BC [corresp ∠'s are equal/*ooreenkomstige ∠e* =] $\frac{AB}{AM} = \frac{AC}{AN}$ [line ‖ one side of Δ **OR** prop theorem; MN ‖BC] $\therefore\ \frac{AB}{DE} = \frac{AC}{DF}$ [AM=DE and AN=DF]

PAPER B

QUESTION 8

8.1.1	$\hat{P} = 116°$	[opp ∠s of cyclic quad/*teenoorst. ∠e van kvh*]
8.1.2	$\hat{M}_1 + 64° = 90°$ $\hat{M}_1 = 26°$	[∠ in semi-circle/∠ *in halwe sirkel*]
8.1.3	$\hat{O}_1 = 52°$	[∠ at centre = 2 x ∠ at circumference/*midpts. ∠ = 2 x omtreks. ∠*]

8.2.1	Midpt theorem/*Midpt. Stelling* **OR/*OF*** Converse prop intercept theorem	
8.2.2	BG = 2DE or $6x - 2$ BG = $6x - 2$ $\frac{GH}{BG} = \frac{FC}{BF}$ $\frac{x+1}{6x-2} = \frac{1}{4}$ $4x + 4 = 6x - 2$ $2x = 6$ $x = 3$ **OR/*OF***	[Midpt theorem/*Midpt. stelling*] [line ‖ one side of Δ **OR** prop theorem; FG ‖ CH / *lyn ‖ een sy v. Δ*]

	$\frac{BF}{FC} = \frac{BG}{GH}$	[line || one side of Δ **OR** prop theorem; FG || CH / *lyn || een sy v.* Δ]
	$\frac{AE}{AG} = \frac{DE}{BG}$	[ΔADE ||| ΔABG]
	$BG = 4x + 4$	
	$\frac{1}{2} = \frac{3x-1}{4x+4}$	
	$\therefore 4x + 4 = 6x - 2$	
	$\therefore x = 3$	

QUESTION 9.2

9.2.1	$O\hat{T}G = 90°$	[line from centre to midpt of chord/ *midpt. sirkel; midpt. koord*]
	$O\hat{B}G = 90°$	[tan ⊥ radius/*raaklyn* ⊥ *radius*]
	$\therefore O\hat{T}G = O\hat{B}G = 90°$	
	$\therefore$ OTBG is a cyclic quadrilateral	[line subtends equal ∠s **OR** converse ∠s in the same segment/ *lyn onderspan gelyke ∠e*]
9.2.2	$\hat{S} = B\hat{T}G$	[corresp ∠s; GF || PS / *ooreenk.* ∠s; GF || PS]
	But $B\hat{T}G = G\hat{O}B$	[∠s in the same segment/∠*e in dies. sirkelsegment*]
	$G\hat{O}B = \hat{S}$	

QUESTION 10

10.1	$\hat{P}_1 = \hat{Q}_1$	[tan-chord theorem/$\angle$ *tussen raaklyn en koord*]
	$\hat{S}_1 = \hat{Q}_1 + \hat{Q}_2$	[ext $\angle$ of cyclic quad/*buite* $\angle$ *v. kvh*]
	$\therefore \hat{S}_1 = \hat{P}_1 + \hat{Q}_2$	
	$\hat{T}_2 = \hat{R}_2 + \hat{Q}_2$	[ext $\angle$ of Δ/*buite* $\angle$ *v.* Δ]
	but $\hat{P}_1 = \hat{R}_2$	[given/*gegee*]
	$\hat{T}_2 = \hat{P}_1 + \hat{Q}_2$	
	$\therefore \hat{S}_1 = \hat{T}_2 = \hat{P}_1 + \hat{Q}_2$	
10.2	In Δ ASD and ΔACR	
	$\hat{A} = \hat{A}$	[common $\angle$/*gemeenskaplike* $\angle$]
	$\hat{S}_1 = \hat{T}_2$	[proven/*reeds bewys*]
	$\hat{T}_2 = \hat{C}_2$]	[alt $\angle$s; QS || CA/verw. $\angle$*e*; QS || CA
	$\therefore \hat{S}_1 = \hat{C}_2$	
	$\hat{D}_1 = \hat{R}_1$	[sum of $\angle$s in Δ/$\angle$*e v.* Δ]
	$\Delta ASD \parallel\parallel \Delta ACR$	
	$\therefore \frac{AD}{AR} = \frac{AS}{AC}$	[corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]
	OR/*OF*	

	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ $\hat{S}_1 = \hat{T}_2$ $\hat{T}_2 = \hat{C}_2$ $\therefore \hat{S}_1 = \hat{C}_2$ $\Delta ASD \mid\mid\mid \Delta ACR$ $\therefore \frac{AD}{AR} = \frac{AS}{AC}$	 [common $\angle$/*gemeenskaplike* $\angle$] [proven/*gegee*] [alt $\angle$s; QS || CA/verw. $\angle$*e*; QS || CA] [$\angle$;$\angle$;$\angle$] [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]
10.3	$\frac{AS}{AC} = \frac{SD}{CR}$ $\therefore AS = \frac{AC \times SD}{CR}$ $\frac{AS}{AR} = \frac{CT}{CR}$ $\therefore AS = \frac{AR \times CT}{CR}$ $\therefore \frac{AC \times SD}{CR} = \frac{AR \times CT}{CR}$ $\therefore AC \times SD = AR \times CT$	[$\Delta ASD \mid\mid\mid \Delta ACR$] [line || one side of Δ OR prop theorem; TS || CA/*lyn* || *een sy v.* Δ]

PAPER C

QUESTION 9

9.

$\hat{C} = 90°$ (∠s in semi circle)

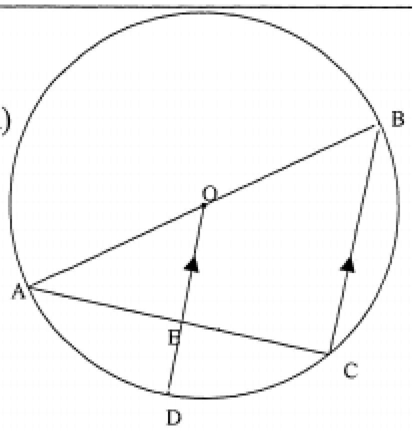
$O\hat{E}A = 90°$ (corres ∠s; OD ∥ BC)

AE = 8 cm (line from circ cent ⊥ ch bis ch)

OE = 6 cm (Pythagoras)

ED = 10 – 6

= 4 cm



OR

$\hat{C} = 90°$ (∠s in semi circle)

$O\hat{E}A = 90°$ (corres ∠s; OD ∥ BC)

OE ∥ BC (given)

OA = OB (radii)

AE = EC = 8cm (midpoint theorem)

OE = 6 cm (Pythagoras)

ED = 10 – 6

= 4 cm

OR

$\hat{C} = 90°$ (∠s in semi circle)

$BC^2 = (20)^2 - (16)^2$

$BC^2 = 144$

BC = 12

$OE = \frac{1}{2}BC$ (midpoint theorem)

OE = 6 cm

OD = 10cm

ED = 10 – 6

= 4 cm

OR

$\hat{C} = 90°$ (∠s in semi circle)

$BC^2 = (20)^2 - (16)^2$

$BC^2 = 144$

BC = 12

$OE = \frac{1}{2}BC$ (midpoint theorem)

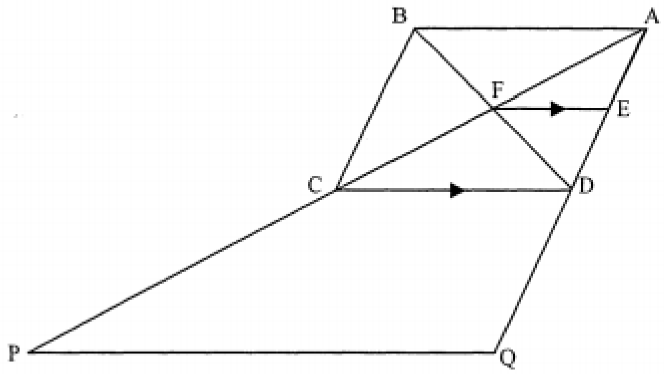
OE = 6 cm

ED = 4cm

Question 10

10.1	$\hat{A} = \hat{D}_4 = x$ (tan ch th) $\hat{E}_2 = x$ (tan ch th) **OR** ($\angle$s in same seg) $\hat{D}_2 = \hat{A} = x$ (alt $\angle$s; CA $\parallel$ DF)
10.2	In ΔBHD and Δ FED 1. $\hat{B}_2 = \hat{F}$ ($\angle$s in same seg) 2. $\hat{D}_3 = \hat{D}_1$ (= chs subt = $\angle$s) ΔBHD $\parallel\!\mid$ Δ FED ($\angle\angle\angle$)
10.3	$\frac{FE}{BH} = \frac{FD}{BD}$ ($\parallel\!\mid$ Δs) But FE = AB (given) $\frac{AB}{BH} = \frac{FD}{BD}$ AB.BD = FD.BH

QUESTION 11



11.1	AF = FC FE ‖ CD AE = ED	(diags of parallelogram bisect) (Prop Th; FE ‖ CD) **OR** (Midpoint Theorem)
11.2	$\frac{AC}{CP}=\frac{1}{2}$ $\frac{AD}{DQ}=\frac{1}{2}$ $\frac{AC}{CP}=\frac{AD}{DQ}$ CD ‖ PQ CD ‖ FE ∴ PQ ‖ FE	(given) (given) (converse proportionality theorem) (given)

	OR $\frac{AC}{AP}=\frac{1}{3}$ $\frac{AD}{AQ}=\frac{1}{3}$ $\frac{AC}{AP}=\frac{AD}{AQ}$ $CD \parallel PQ$ (converse proportionality theorem) $CD \parallel FE$ (given) $\therefore PQ \parallel FE$ **OR** $\frac{AF}{AP}=\frac{1}{6}$ $\frac{AE}{AQ}=\frac{1}{6}$ $\frac{AF}{AP}=\frac{AE}{AQ}$ $\therefore PQ \parallel FE$ (converse proportionality theorem)

11.3	In ΔAEF and ΔAPQ 1. $\hat{A}$ is common 2. $A\hat{E}F = A\hat{Q}P$ (corres $\angle$s; $FE \parallel PQ$) 3. $A\hat{F}E = A\hat{P}Q$ (corres $\angle$s; $FE \parallel PQ$) $\therefore \Delta AEF \parallel\!\mid \Delta AQP$ ($\angle\angle\angle$) $\frac{FE}{PQ}=\frac{AF}{AP}$ ($\parallel\!\mid \Delta$s) $\frac{FE}{60}=\frac{1}{6}$ $FE = 10$ cm

OR

In ΔADC and ΔAPQ

1. $\hat{A}$ is common
2. $A\hat{D}C = A\hat{Q}P$ (corres $\angle$s; CD || PQ)
3. $A\hat{C}D = A\hat{P}Q$ (corres $\angle$s; CD || PQ)

$\therefore \Delta ADC \parallel\!| \Delta AQP$ ($\angle\angle\angle$)

$\frac{AC}{AP} = \frac{AD}{AQ} = \frac{1}{3}$ (||| Δs)

$CD = \frac{1}{3}PQ$

$CD = 20$ cm

But AF = FC

AE = ED (Midpoint Theorem)

$FE = \frac{1}{2}CD$

$FE = 10$ cm

PAPER D

QUESTION 8

8.1.1	$M\hat{R}P = 90°$ $\hat{R}_2 = 21°$	[$\angle$ in semi circle/$\angle$ *in halwe sirkel*]
8.1.2	$\hat{O}_1 = 138°$	[$\angle$ at centre = 2 $\times\angle$ at circumference/ *midpts.* $\angle = 2\times$ *omtreks* $\angle$]
8.1.3	$\hat{M}_1 = 21°$ **OR** $\hat{M}_1 + N_1 = 180° - 138°$ $\therefore \hat{M}_1 = 21°$	[$\angle$s in the same segment/$\angle$*e in dieselfde sirkel segment*] [sum of $\angle$s in Δ/$\angle$*e v* Δ] [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*]
8.1.4	$\hat{O}_2 = 42°$ $\hat{P} = 42°$ $\hat{M}_2 = 48°$ **OR** $\hat{N}_2 = \hat{R}_2 = 21°$ $\hat{N}_1 = \hat{M}_1 = 21°$ $\hat{M}_2 = 48°$	[$\angle$s on a str line/$\angle$*e op 'n reguitlyn*] [alt $\angle$s; NO || PR/*Verw.* $\angle$*e*, NO || PR] [sum of $\angle$s in Δ/$\angle$*e v* Δ] [alt $\angle$s; NO || PR/*Verw.* $\angle$*e*, NO || PR] [$\angle$s opposite equal sides/$\angle$*e teenoor gelyke sye*] [sum of $\angle$s of Δ NMR//$\angle$*e v* ΔNMR]

8.2	$\hat{D}_1 = 4x$ [ext $\angle$ of Δ/*buite* $\angle$ *v* Δ] $\hat{D}_2 = 180° - 4x$ [$\angle$s on a str line/$\angle$*e op 'n reguitlyn*] $\hat{B}_1 = 5x$ [ext $\angle$ of Δ/*buite* $\angle$ *v* Δ] $\hat{B}_1 = \hat{D}_2$ [ext $\angle$ of cyclic quad/*buite* $\angle$ *v kvh*] $180° - 4x = 5x$ $9x = 180°$ $x = 20°$ **OR** $\hat{C}_1 = 3x$ [ext $\angle$ of cyclic quad/*buite* $\angle$ *v kvh*] $\hat{B}_2 = 4x$ [ext $\angle$ of Δ/*buite* $\angle$ *v* Δ] $\hat{C}_1 = \hat{C}_3 = 3x$ [vert opp $\angle$s] $\hat{D}_2 = 5x$ [ext $\angle$ of Δ/*buite* $\angle$ *v* Δ] $4x + 5x = 180°$ [opp $\angle$ of cyclic quad/*teenoorst.* $\angle$*e v kvh*] $x = 20°$

	OR $\hat{C}_3 = 3x$ [ext $\angle$ of cyclic quad/*buite* $\angle$ *v kvh*] $\hat{D}_1 = 4x$ [ext $\angle$ of Δ/*buite* $\angle$ *v* Δ] $2x + 3x + 4x = 180°$ [sum of $\angle$s in Δ/$\angle$*e v* Δ] $9x = 180°$ $x = 20°$

QUESTION 9.2

9.2.1	$\hat{A}_1 = x$ [corresp $\angle$s; PQ ‖ CA/*ooreenkomstige* $\angle$*e,* PQ ‖ CA] $\hat{B} = x$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*] $\hat{A}_2 = x$ [tan-chord theorem/$\angle$ *tussen raaklyn en koord*] $\hat{P} = x$ [alt $\angle$s; PQ ‖ CA/*verw.* $\angle$*e,* PQ ‖ CA]
9.2.2	$\hat{B} = \hat{P}$ [proved in 9.2.1/*bewys in 9.2.1*] $\therefore$ A, B, P and R are concyclic $\therefore$ ABPR is a cyclic quadrilateral [conv $\angle$s in the same segment/ *koord onderspan gelyke omtreks* $\angle$*e*]
9.2.3	$\frac{BA}{BQ} = \frac{BC}{BR}$ [prop th; AC ‖ QP] **OR** [line ‖ one side Δ/*lyn* ‖ *een syn v* Δ] But QR = BR [sides opp = $\angle$s/*sye teenoor* = $\angle$*e*] $\therefore \frac{BA}{BQ} = \frac{BC}{QR}$

OR

In Δ ABC and Δ BQR:

$\hat{A}_1 = \hat{B} = x$ [proved in 9.2.1]

$\hat{B} = \hat{Q} = x$ [proved in 9.2.1]

$\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ]

$\therefore \Delta ABC \parallel\mid \Delta BQR$

$\therefore \frac{BA}{BQ} = \frac{BC}{QR}$

OR

In Δ ABC and Δ BQR:

$\hat{A}_1 = \hat{B} = x$ [proved in 9.2.1]

$\hat{B} = \hat{Q} = x$ [proved in 9.2.1]

$\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ]

$\therefore \Delta ABC \parallel\mid \Delta BQR$ [$\angle\angle\angle$]

$\therefore \frac{BA}{BQ} = \frac{BC}{QR}$

OR

In Δ ABC and Δ QBR:

$\hat{B}$ is common

$\hat{A}_1 = \hat{Q} = x$ [corres $\angle$s; PQ || CA]

$\hat{C}_1 = B\hat{R}Q = 180° - 2x$ [sum of $\angle$s of Δ]

$\therefore \Delta ABC \parallel\mid \Delta QBR$ [$\angle\angle\angle$]

But QR = BR [sides opp = $\angle$s/*sye teenoor* = $\angle$*e*]

$\therefore \frac{BA}{BQ} = \frac{BC}{QR}$

QUESTION 10

10.1.1	$\hat{Q}_1 + \hat{Q}_2 = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $\therefore \hat{M}_2 = 90°$ [co-interior ∠, MS ‖ QR/*ko-binne ∠e*, MS ‖ QR] $\therefore$ SQ is a diameter [converse: ∠ in semi circle/ *Omgekeerde: ∠ in halwe sirkel*] **OR** MS ‖ QR $\frac{TS}{SR} = \frac{TM}{MQ} = \frac{1}{1}$ [prop theorem; SM ‖ QR] **OR** [line ‖ one side of Δ]/*lyn ‖ een sy v*Δ $\therefore TM = MQ$ $\therefore \hat{M}_2 = 90°$ [Line from centre bisects chord/*midpt. sirkel; midpt koord*] $\therefore$ SQ is a diameter [converse: ∠ in semi circle/ *Omgekeerde: ∠ in halwe sirkel*] **OR** $SQ \perp QP$ [tan ⊥ rad/*raaklyn ⊥ radius*] $\therefore$ SQ is a diameter [converse: tan ⊥ rad/*Omgekeerde: raaklyn ⊥ radius*]
10.1.2	In ΔRTQ and ΔRQP $\hat{T} = \hat{Q}_3$ [tan-chord theorem/∠ *tussen raaklyn en koord*] $\hat{Q}_1 + \hat{Q}_2 = 90°$ [co-interior ∠s, MS ‖ QR/*ko-binne ∠e*, MS ‖ QR] **or** [∠ in semi circle/∠ *in halwe sirkel*] $\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{P} = 90°$ $\hat{R}_1 = \hat{R}_2$ [∠s of Δ/∠*e van Δ*] $\Delta RTQ \parallel\!\mid \Delta RQP$ $\frac{RT}{RQ} = \frac{RQ}{RP}$ $RT = \frac{RQ^2}{RP}$

	OR	
	In ΔRTQ and ΔRQP	
	$\hat{T} = \hat{Q}_3$	[tan-chord theorem $\angle$ *tussen raaklyn en koord*]
	$\hat{Q}_1 + \hat{Q}_2 = 90°$	[co-interior $\angle$s, MS ∥ QR/*ko-binne* $\angle$*e*, MS ∥ QR] **or** [$\angle$ in semi circle/$\angle$ *in halwe sirkel*]
	$\therefore \hat{Q}_1 + \hat{Q}_2 = \hat{P} = 90°$	
	$\Delta RTQ \parallel\!\mid \Delta RQP$	[$\angle,\angle,\angle$]
	$\frac{RT}{RQ} = \frac{RQ}{RP}$ $RT = \frac{RQ^2}{RP}$	
10.2	$QR = 28$ units	[midpoint theorem/*midpt. stelling*]
	$RP^2 = 28^2 - \left(\sqrt{640}\right)^2$	[Pythagoras/*Pythagoras*]
	$RP = 12$ units	
	$RT = \frac{RQ^2}{RP}$ $RT = \frac{28^2}{12}$ $RT = \frac{196}{3}$ Radius $= \frac{98}{3}$ units	

PAPER E

QUESTION 9

9.1	tangents from same(common) point/*raaklyne vanaf dieselfde punt*
9.2.1	$\hat{S}_2 = S\hat{R}T$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*] $\therefore \hat{S}_2 = 51°$ [sum of $\angle$s in Δ/*som van* $\angle$*e in* Δ]
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93°$ [ext $\angle$ of cyclic quad/*buite*$\angle$ *van koordevh*] $\hat{S}_3 = 42°$ **OR/*OF*** $\hat{S}_1 = 87°$ [opp $\angle$s of cyclic quad/*teenoorst* $\angle$*e v kdvh*] $\hat{S}_3 = 180° - (87° + 51°)$ $\hat{S}_3 = 42°$ [$\angle$s on a str line/$\angle$*e op reguitlyn*]

QUESTION 10

10.1	line from centre ⊥ to chord/*lyn vanaf middelpunt* ⊥ *op koord*
10.2	$\therefore \hat{A}_1 = 90° - x$ [sum of ∠s in Δ/*som van* ∠*e in* Δ] $\therefore \hat{M}_1 = 180° - 2x$ [∠at centre=2× at circumf/*midpts*∠= 2×omtreks∠]
10.3	$C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] $\hat{A}_2 = 90° - (90° - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore$ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $E\hat{M}D = 2x$ [adj suppl ∠s/*aanligg suppl* ∠*e*] $\therefore \hat{A}_2 = x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $\therefore \hat{A}_2 = C = x$ $\therefore$ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $\hat{M}_3 = 180° - 2x$ [vert. opp/ *regoorstaande* ∠*e*] $\therefore \hat{A}_3 = 90° - x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $B\hat{A}E = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_2 = C = x$ $\therefore$ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF***

	$CD \parallel AB$ [midpt. Thm/ *middelpuntst.*] $B\hat{A}E = 90^\circ$ [$\angle$ in semi-circle/$\angle$ *in halfsirkel*] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt.$\angle$s; CD$\parallel$AB/*verwiss* $\angle$*e*] $\therefore \hat{A}_2 = x = C$ $\therefore$ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $C\hat{A}D = 90°$ [$\angle$ in semi circle/$\angle$ *in halfsirkel*] AC = diameter [converse $\angle$ in semi circle/*omgek* $\angle$ *in halfsirkel*] $\therefore$ AD is a tangent [converse radius $\perp$ tangent/*omgek radius*$\perp$*rkl*]
10.4	AF = FE and BM = ME [given & radii] $\therefore FM = \frac{1}{2}AB = 12$ units [Midpt Theorem/*middelptstelling*] EM = MB = CM = 18 units [radii] $\therefore$ EB = 36 units [diameter = 2 radius] $\therefore AE^2 = (36)^2 - (24)^2$ [Pythagoras] $AE = 12\sqrt{5}$ or 26,83 units **OR/*OF*** AF = FE and BM = ME [given & radii] $\therefore FM = \frac{1}{2}AB = 12$ units [Midpt Theorem/*middelpuntstelling*] EM = MB = CM = 18 units [radii] $\therefore FE^2 = (18)^2 - (12)^2$ [Pythagoras] $FE = 6\sqrt{5}$ $AE = 12\sqrt{5}$ or 26,83 units

QUESTION 11.2

11.2.1(a)

$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/*raaklynkoordstelling*]

$\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp $\angle$s; / *ooreenk* $\angle$*s;* MN || KS]

$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$

11.2.1(b)

$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$

$\therefore$ KLMN is a cyclic quad [ext $\angle$ of quad = opp int $\angle$ / *buite* $\angle$ *van vh = teenoorst binne* $\angle$]

OR/OF

$N_1 = \hat{K}_1 + \hat{K}_2 = N\hat{K}S$ [corresp $\angle$s; / *ooreenk* $\angle$*s;* MN || KS]

$N\hat{K}S = K\hat{L}S$ [tan chord theorem / *raaklynkoordstelling*]

$\hat{N}_1 = K\hat{L}S$

$\therefore$ KLMN is a cyclic quad [ext $\angle$ of quad = opp int $\angle$ / *buite* $\angle$ *van vh = teenoorst binne* $\angle$]

OR/OF

$NKL = 180^\circ - K_4$ [adj. suppl.]

$\therefore NKL = 180^\circ - NML$ [proved]

$\therefore$ KLMN is a cyclic quad [opp.$\angle$s supplementary]

11.2.2

In $\Delta LKN \,|||\, \Delta KSM$:

$\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / $\angle$*e in dieselfde sirkel segm*]

$\hat{L}_1 = \hat{M}_2$ [$\angle$s in the same seg / $\angle$*e in dieselfde sirkel segm*]

$= \hat{K}_2$ [alt $\angle$s; / *verw*$\angle$*e;* MN||KS]

$N\hat{K}L = M\hat{S}K$ [$\angle$s of Δ / $\angle$*e van* Δ]

$\Delta LKN \,|||\, \Delta KSM$

	OR/OF In $\Delta LKN \parallel\!\mid \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / $\angle$*e in dieselfde sirkel segm*] $N\hat{K}L = \hat{M}_1$ [ext $\angle$ of cyclic quad/*buite*$\angle$ *van koordevh*] $= \hat{S}_2$ [corresp $\angle$s/*ooreenk* $\angle$*e*; KS || NM] $\Delta LKN \parallel\!\mid \Delta KSM$ [$\angle,\angle,\angle$] **OR/OF** In $\Delta LKN \parallel\!\mid \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / $\angle$*e in dieselfde sirkel segm*] $\hat{K}_4 + N\hat{K}L = \hat{S}_1 + \hat{S}_2$ [$\angle$s on straight line/$\angle$*e op reguitlyn*] $\therefore\ N\hat{K}L = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\Delta LKN \parallel\!\mid \Delta KSM$ [$\angle,\angle,\angle$]
11.2.3	$\frac{LK}{KS} = \frac{KN}{SM}$ [$\Delta LKN \parallel\!\mid \Delta KSM$] $\therefore \frac{12}{KS} = \frac{4}{3}$ $KS = 9$ units
11.2.4	$4SM = 3KN$ $SM = \frac{3(8)}{4}$ $SM = 6$ $\frac{LT}{NL} = \frac{LS}{ML}$ [line || one side of Δ / *lyn* || *een sy v* Δ] $\frac{LT}{16} = \frac{13}{19}$ $LT = \frac{208}{19} = 10{,}95$

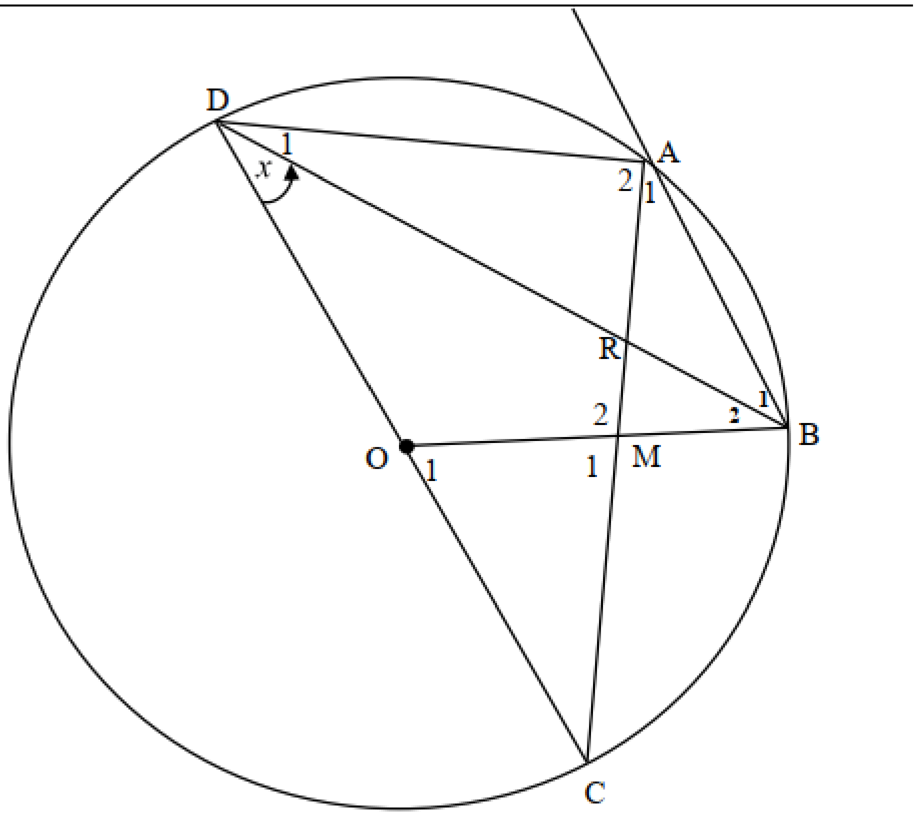
PAPER F

8.1.1	$A\hat{D}C = 67°$ **OR / *OF*** $\hat{B}_2 + \hat{B}_3 = 113°$ $A\hat{D}C = 67°$	ext. ∠ of cyclic quad / *buite ∠ van kvh* ∠s straight line / ∠e *op reguit lyn* opp ∠s of cyclic quad / *oorst ∠e van kvh*
8.1.2	$\hat{C} = 180° - 67°$ $= 113°$	co-int ∠s BC || AD / *ko-binne ∠e BC || AD*
8.1.3	$\hat{A} = 67°$	opp ∠s of cyclic quad / alt ∠s BC||AD / alt ∠s EC || AD *oorst ∠e van kvh / verwisselende ∠e* *BC||AD / verwis ∠e EC || AD*
8.1.4	$\hat{B}_2 = 67°$ $\hat{D}_2 = 180° - 67° - 67°$ $= 46°$	∠s opposite = sides / ∠e *teenoor = sye* sum of ∠s in Δ / *som vd ∠e v Δ*
8.1.5	$B\hat{D}G = 113°$ **OR / *OF*** $\hat{D}_1 = 67°$ $B\hat{D}G = 113°$	tan chord theorem / *raaklyn koordstelling* tan chord theorem / *raaklyn koordstelling*

8.2	$\hat{B}_3 = \hat{D}_2 = 46°$ $AB = CD$	alt ∠s BC||AD / *verwisselende ∠e* BC||AD ∠s subtend = chords / ∠e *onderspan = koorde*

QUESTION / *VRAAG* 9

9.1

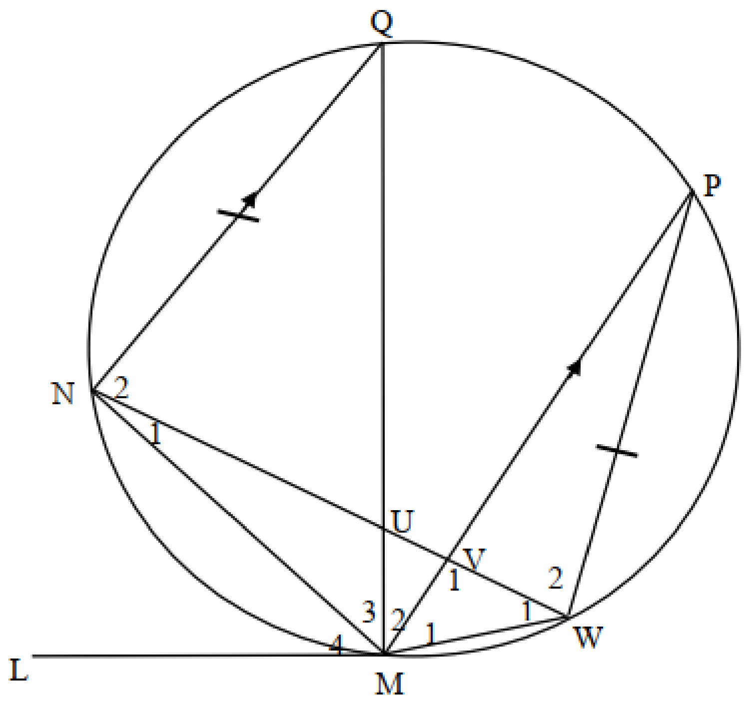


9.1.1(a)	$\hat{O}_1 = 2x$	$\angle$ centre $= 2\times\angle$ circumference / *middelpunts $\angle = 2\times$ omtreks $\angle$*

9.1.1(b)	$\hat{A}_1 = C\hat{D}B = x$	$\angle^s$ in the same segment / *$\angle^e$ in dies. segment*
	$\hat{M}_2 = 90°$	line from centre to midpoint of chord / *lyn van middelpunt van sirkel na middelpunt van koord*
	$\therefore A\hat{B}O = 90° - x$	sum of $\angle^s$ in Δ / ext $\angle$ of a Δ / *som vd $\angle^e$ v Δ / buite $\angle$ v Δ*
	OR/*OF*	
	$\hat{O}_1 = 2x$	proved/*reeds bewys*
	$\hat{M}_1 = 90°$	line from centre to midpoint of chord / *lyn van middelpunt van sirkel na middelpunt van koord*
	$\hat{C} = 90° - 2x$	sum of $\angle^s$ in Δ / *som vd $\angle^e$ v Δ*
	$\hat{B}_1 = 90° - 2x$	$\angle^s$ in the same segment / *$\angle^e$ in dies. segment*
	$A\hat{B}O = 90° - x$	

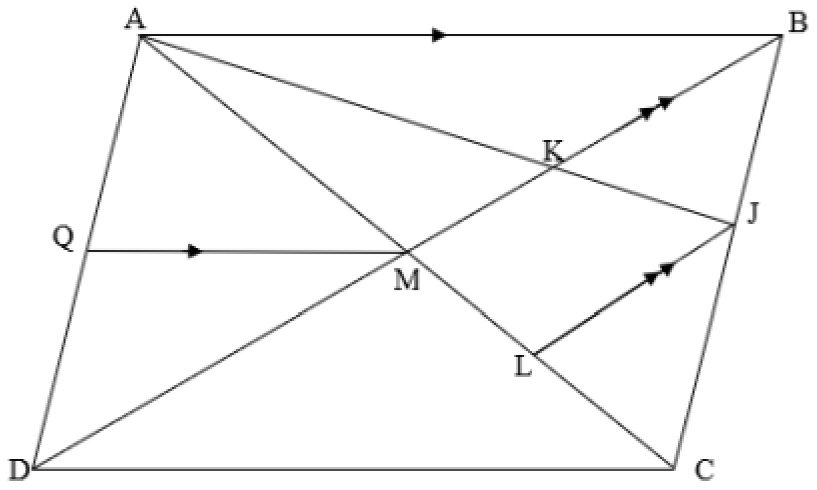
9.1.2	$AD \parallel OB$ $\hat{O}_1 = A\hat{D}C = 2x$ $\therefore \hat{D}_1 = x$ $\hat{A}_1 = x$ $\therefore \hat{D}_1 = \hat{A}_1$ AB is a tangent / *is 'n raaklyn* **OR / *OF*** $\hat{A}_2 = 90°$ $\therefore AD \parallel OB$ $C\hat{D}A = \hat{O}_1 = 2x$ $\therefore \hat{D}_1 = x$ $\hat{D}_1 = \hat{A}_1$ AB is a tangent / *is 'n raaklyn*	midpoint theorem / *middelpunt stelling* corresponding $\angle^s$ AD || OB / *ooreenkom $\angle^e$ AD || OB* proved / *reeds bewys* converse tan chord theorem / *omgekeerde raaklyn koordstelling* $\angle$ in a semi-circle / *$\angle$ in halwe sirkel* corr $\angle^s$ are equal / *ooreenk $\angle^e$ gelyk* corr $\angle^s$ DA || OB / *ooreenk $\angle^e$* DA || OB converse tan chord theorem / *omgekeerde raaklyn koord*
9.1.3	$DC^2 = AD^2 + AC^2$ but / *maar* $AC = 2AM$ and / *en* $DC = 2DO$ $(2DO)^2 = AD^2 + (2AM)^2$ $4DO^2 = AD^2 + 4AM^2$ but / *maar* In ΔABM $AM^2 = AB^2 - MB^2$ $\therefore 4DO^2 = AD^2 + 4(AB^2 - MB^2)$ $AD^2 = 4DO^2 - 4AB^2 + 4MB^2$	Pythagoras Pythagoras

9.2



9.2.1	$\hat{M}_2 = \hat{Q}$	alternate $\angle^s$ NQ || MP / *verwissel* $\angle^e$ *NQ ||MP*
	$\hat{M}_4 = \hat{Q} = \hat{M}_2$	tan chord theorem / *raaklyn koordstelling*
	$\hat{W}_1 = \hat{Q} = \hat{M}_2$	$\angle^s$ in the same segment / tan chord theorem / $\angle^e$ *dieselfde sirkel segment / raaklyn koord stelling*
	OR/*OF*	
	$\hat{W}_1 = \hat{M}_4 = \hat{M}_2$	tan chord theorem / *raaklyn koordstelling*
9.2.2	In ΔWMV and/*en* ΔQMN	
	$\hat{W}_1 = \hat{Q}$	proved / *reeds bewys*
	$\hat{M}_1 = \hat{M}_3$	equal chords subtend equal $\angle^s$ / *gelyke koorde onderspan gelyke* $\angle^e$
	$\hat{V}_1 = \hat{N}_1 + \hat{N}_2$	sum of $\angle^s$ in Δ / *som vd* $\angle^e$ *v* Δ
	$\therefore \Delta WMV \parallel\!\mid \Delta QMN$	$\angle\angle\angle$

9.2.3	$\frac{MV}{MN}=\frac{WV}{QN}$	$\Delta WMV \,\|\|\|\, \Delta QMN$ / $\|\|\| \,\Delta^s$
	$\frac{MV}{WV}=\frac{MN}{QN}$	
	$MV \times QN = MN \times WV$	given / *gegee*
	but / *maar* $QN = PW$	
	$MV \times PW = MN \times WV$	
	$\frac{MV}{WV}=\frac{MN}{PW}$	

10.2		
10.2.1 (a)	$\frac{ML}{LC}=\frac{BJ}{JC}=\frac{2}{3}$	line || one side ΔBCM **OR** prop theorem MB||JL / *lyn || aan een sy van* ΔBCM ***OF*** *eweredigheidsstelling MB||JL*

10.2.1 (b)	$\frac{MC}{ML}=\frac{BC}{BJ}=\frac{5}{2}$	line || one side ΔBMC **OR** prop theorem MB||JL / *lyn || aan een sy van* ΔBMC ***OF*** *eweredigheidsstelling MB || JL*
	$AM = MC$	diagonals of a parm bisect / *hoeklyne van parm halveer*
	$\frac{AM}{ML}=\frac{5}{2}$	
	$\frac{AK}{KJ}=\frac{AM}{ML}=\frac{5}{2}$	line || one side ΔAJL **OR** prop theorem MK||JL / *lyn || aan een sy van* ΔAJL ***OF*** *eweredigheidsstelling MK || JL*

10.2.2	AB ‖ CD AB ‖ QM In ΔADC $\therefore$ QM ‖ CD AM = MC $\therefore$ AQ = QD *but* AD = BC $AQ = \frac{1}{2}AD$ $= \frac{1}{2}\left(\frac{2\sqrt{10}}{3}\right)$ $\therefore AQ = QD = \frac{2}{3}\sqrt{10} \div 2$ $= \frac{\sqrt{10}}{3}$ units	opposite sides of parm / *oorst sye van parm* proved / *reeds bewys* line passing through the midpoint of 1 side ‖ to second side / *lyn sny die middelpunt van 1 sy ‖ aan tweede sy* opposite sides of parm / *oorst sye van parm*

PAPER G

8.1	$\hat{O}_2 = 50^\circ$ $\hat{D}_1 = 25^\circ$	$\angle$s around a point / $\angle$*e om 'n punt* $\angle$ centre = 2 × $\angle$ at circumference *midpts* $\angle$ = 2 × *omtreks* $\angle$
8.2	$\hat{B}_3 = 25^\circ$	tan chord theorem / *raaklyn koordstelling*
8.3	$B\hat{C}D = 120^\circ$ $\hat{B}_2 = 35^\circ$ $O\hat{B}C = O\hat{C}B = 65^\circ$ $\therefore \hat{B}_1 = 65^\circ - 35^\circ$ $\hat{B}_1 = 30^\circ$ **OR / *OF*** $B\hat{C}D = 120^\circ$ $\hat{B}_2 = 35^\circ$ $\hat{B}_1 + \hat{B}_2 + \hat{B}_3 = 90^\circ$ $\hat{B}_1 = 30^\circ$	opp $\angle$s of a cyclic quad /*teenoorst* $\angle$*e v kvh* sum of $\angle$s of a triangle / *som* $\angle$*e v* Δ $\angle$s opp. equal radii / $\angle$*e teenoor gelyke radiuse* **OR / *OF*** opp $\angle$s of a cyclic quad/*teenoorst* $\angle$*e vkvh* sum of $\angle$s of a triangle / *som* $\angle$*e v* Δ radius $\perp$ tangent / *radius* $\perp$ *raaklyn*

QUESTION / *VRAAG* 9

9.1	Equal / *gelyk.*	
9.2.1	$\hat{D}_1 = \hat{F}_1 = x$	tan chord theorem / *raaklyn koordstelling*
	$\hat{F}_1 = \hat{F}_2 = x$	= chords subtend = $\angle s$ = *koorde onderspan* = $\angle e$
9.2.2	$\hat{F}_2 = \hat{A} = x$	ext. $\angle$ of cyclic quad / *buite* $\angle$ *v kvh*
	$\hat{D}_1 = \hat{A} = x$	
	ABDC is a cyclic quad / *ABDC is 'n kvh*	ext $\angle$ = opp int $\angle$ **OR** converse of ext. $\angle$ of cyclic quad / *buite* $\angle$ = *oorst binne* $\angle$ ***OF*** *omgekeerde buite* $\angle$ *v kvh*
9.2.3	$\hat{B}_1 + \hat{B}_2 = \hat{A}$	tan chord theorem / *raaklyn koordstelling*
	$\hat{A} = \hat{D}_1$	proved / *reeds bewys*
	$\hat{B}_1 + \hat{B}_2 = \hat{D}_1$	
	BE || CD	correspond $\angle s$ = / *ooreenkomst* $\angle e$ =

9.2.4	$\hat{C}_1 + \hat{C}_2 + \hat{F}_1 + \hat{F}_2 = 180°$	opp $\angle s$ of a cyclic quad / *teenoorst* $\angle e$ *v kvh*
	$\hat{C}_1 = \hat{C}_2$	diag rhombus bisect $\angle$ / *diag ruit halveer* $\angle$
	$\hat{F}_1 = \hat{F}_2$	proved / *reeds bewys*
	$2\hat{C}_1 + 2\hat{F}_2 = 180°$	
	$\hat{C}_1 + \hat{F}_2 = 90°$	
	$\hat{E}_1 = 90°$	sum of $\angle s$ of Δ / *som van* $\angle e$ *v* Δ
	FC is a diameter of circle FDCE. *FC is 'n middellyn van sirkel FDCE.*	converse $\angle$ in a semi circle / *omgekeerde* $\angle$ *in half sirkel*

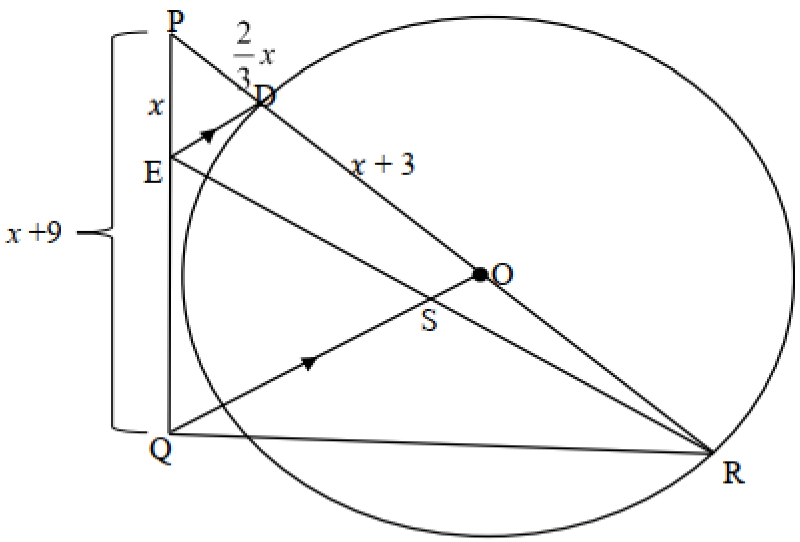
	OR / *OF*	
	Let $\hat{F}_1 = \hat{F}_2 = x$	proved / *reeds bewys*
	$\hat{C} = 180° - 2x$	opp $\angle$s of a cyclic quad / *teenoorst $\angle$e v kvh*
	$\hat{C}_1 = \hat{C}_2 = 90° - x$	diag rhombus bisect $\angle$ / *diag ruit halveer $\angle$*
	In ΔFDC **or / *of*** ΔEFC	
	$\hat{D} = 90°$ **or / *of*** $\hat{E} = 90°$	sum of $\angle$s of Δ / *som van $\angle$e v Δ*
	FC is a diameter of circle FDCE. *FC is 'n middellyn van sirkel FDCE.*	converse $\angle$ in a semi circle / *omgekeerde $\angle$ in half sirkel*

10.2

10.2.1	$\hat{R}_1 = 60°$	equilateral Δ / *gelyksydige Δ*
	$\hat{W}_1 = \hat{P}_1 + \hat{Q}_1$	ext. $\angle$ of a Δ / *buite $\angle$ v Δ*
	$= 60° + \hat{Q}_1$	
	$= \hat{R}_1 + \hat{Q}_1$	
	$\hat{Q}_1 = \hat{R}_2$	$\angle$s in the same segment/ *$\angle$e in dieselfde segment*
	$\therefore \hat{W}_1 = T\hat{R}Q$	
10.2.2	In ΔTQR and / *en* ΔQRV	
	1. $\hat{W}_1 = T\hat{R}Q$	proved / *reeds bewys*
	2. $\hat{R}_1 = T\hat{Q}R$	equilateral Δ / *gelyksydige Δ*
	3. $\hat{Q}_2 = \hat{T}$	sum $\angle$s of Δ / *som van $\angle$e v Δ*
	$\therefore \Delta WRQ \,\|\|\!\|\, \Delta RQT$	$\angle\angle\angle$

10.2.3	In ΔTPV and / *en* ΔWQR 1. $P\hat{Q}R = \hat{R}_1$ both 60° / *albei* 60° $P\hat{Q}R = \hat{V}_1$ ext.∠ of a cyclic quad. / *buite∠ v kvh* $\hat{V}_1 = \hat{R}_1$ 2. $\hat{P}_2 = T\hat{R}Q$ ext.∠ of a cyclic quad. / *buite∠ v kvh* but / *maar* $\hat{W}_1 = T\hat{R}Q$ proved / *reedsbewys* $\hat{P}_2 = \hat{W}_1$ 3. $\hat{T} = \hat{Q}_2$ sum of ∠s of Δ / *som v ∠e v Δ* $\Delta VPT \parallel\!\mid \Delta RWQ$ ∠∠∠ $\frac{VP}{RW} = \frac{PT}{WQ} = \frac{VT}{RQ}$ corresponding sides in prop/ *ooreenkomstige sye in verhouding* $\therefore \frac{PT}{WQ} = \frac{PV}{WR}$

QUESTION / *VRAAG* 11



11.1	$\frac{PE}{EQ}=\frac{PD}{DO}$	line ∥ one side ΔPOQ **OR** prop theorem ED∥OQ / *lyn ∥ een sy* ΔPOQ ***OF*** *eweredigheid stelling ED∥OQ*
	$\frac{x}{9}=\frac{\frac{2}{3}x}{x+3}$	
	$x^2+3x=6x$	
	$x^2-3x=0$	
	$x(x-3)=0$	
	$x=0$ or/*of* $x=3$	
	N.A / *n.v.t*	
	$DO=6$	
	$DO=OR$	radii / *radiusse*
	$OR=6$ units / *eenhede*	

11.2	S is the midpoint of RE / *S is die middelpunt van RE*	
	DE =2OS	midpoint theorem / *middelpunt stelling*
	DE = 2,8 units / *eenhede*	
11.3	$\frac{\text{Area }\Delta PED}{\text{Area }\Delta PER}=\frac{PD}{PR}$	same height (DE) / *dieselfde hoogte (DE)*
	$=\frac{2}{14}$	
	$=\frac{1}{7}$	
	Area $\Delta PER = 7 \times$ Area ΔPED	
	$= 18,9$ units^2 / *eenhede*^{2}	

PAPER H

QUESTION 9

9.1.1	$D\hat{G}F = \hat{E}_4 = 72°$ [ext ∠ of cyclic quad/ *buite ∠ v kvh*]
9.1.2	$\hat{G}_2 = 72° - 16° = 56°$ $\hat{T} = \hat{G}_2 = 56°$ [∠s in the same seg/ *∠e in dies. ⊙ segment*]
9.1.3	$\hat{F}_1 = \hat{E}_4 = 72°$ [alt ∠s; DE ∥ GF / *verw. ∠e*; DE ∥ GF] $\therefore G\hat{E}F = 52°$ [sum of ∠s in Δ / *∠e van* Δ] **OR/*OF*** $\hat{E}_1 = 56°$ [alt ∠s; DE ∥ GF / *verw. ∠e*; DE ∥ GF] $\therefore G\hat{E}F = 52°$ [∠s on a str. line/ *∠e op 'n reguitlyn*]

9.2.1	$NP = PL = 16$ [diag of ∥m / *hoeklyne van ∥m*] $PT = 4$ $NP : PT = 16 : 4$ $= 4 : 1$
9.2.2	$NM : MS = 4 : 1$ $NP : PT = NM : MS$ KM ∥ RS [line divides two sides of Δ in prop / *Lyn verdeel 2 sye v* Δ *eweredig*] **OR/*OF*** [converse prop theorem / *omgekeerde lyn ∥ een sy v* Δ]
9.2.3	$\frac{RL}{KL} = \frac{TL}{LP}$ [prop theorem; KM ∥ RS **OR** line ∥ one side of Δ / *Lyn ∥ een sy v* Δ] $RL = \frac{12 \times 21}{16}$ $= 15{,}75$

	OR / *OF* $NM : MS = 4 : 1$ $KR = MS = 5{,}25$ [opp side of $\|\|^{m}$ / *teenoorst. sye van* $\|\|^{m}$] $KL = NM = 21$ $RL + 5{,}25 = 21$ $RL = 15{,}75$

QUESTION 10.2

10.2.1(a)	$F\hat{C}O = 90°$ [tan ⊥ radius / *raaklyn* ⊥ *radius*] $\hat{F}_1 = 90°$ [BF ⊥ EC] $\therefore F\hat{C}O = \hat{F}_1 = 90°$ $FB \parallel CG$ [corresp ∠s = / *ooreenk.* ∠ *gelyk*]
10.2.1(b)	In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / ∠ $\frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] $\hat{B}_2 = \hat{B}_3$ [sum of ∠s in Δ / ∠*e van* Δ] $\therefore \Delta FCB \parallel\!\mid \Delta CDB$ **OR/*OF*** In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / ∠ $\frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] $\therefore \Delta FCB \parallel\!\mid \Delta CDB$ [∠,∠,∠]

10.2.2	$\hat{G}_1 = 90°$	[line from centre to midpt of chord / *midpt.* ⊙; *midpt. koord*]
10.2.3	In ΔGCD and ΔCDB $\hat{G}_2 = B\hat{C}D = 90°$ $\hat{C}_3 = \hat{D}_2$ $G\hat{D}C = \hat{B}_3$ $\therefore \Delta GCD \parallel\!\mid \Delta CDB$ $[\angle,\angle,\angle]$ $\therefore \dfrac{CD}{DB} = \dfrac{CG}{CD}$ $\therefore CD^2 = CG.DB$	 [$\angle$s opp equal sides / $\angle$*e teenoor gelyke sye*] [sum of $\angle$s in Δ / $\angle$*e van* Δ] [$\parallel\!\mid$ Δs]
10.2.4	$\dfrac{BC}{DB} = \dfrac{FB}{BC}$ $\therefore BC^2 = DB.FB$ $CD^2 + BC^2 = CG.DB + DB.FB$ $DB^2 = DB(CG + FB)$ $DB = CG + FB$	[$\Delta FCB \parallel\!\mid \Delta CDB$]

PAPER I

QUESTION 8

8.1.1(a)	$\hat{T}_2 = 54°$	[tan ⊥ rad]
8.1.1(b)	$\hat{L} = 36°$	[tan - chord theorem]
8.1.1(c)	$K\hat{O}T = 72°$	[∠ at centre = 2× ∠ at circumference]
	OR/*OF*	
	$O\hat{K}T = \hat{T}_2 = 54°$	[∠s opposite = radii]
	$K\hat{O}T = 180° - (54° + 54°)$ $= 72°$	[sum of int ∠'s of Δ]
8.1.2	$K\hat{M}O = 180° - (18° + 72°)$ $= 90°$	[sum of int ∠'s of Δ]
	$\therefore$ KM = ML	[line from centre ⊥ to chord]
	OR/*OF*	
	$O\hat{K}T = 54°$	[∠s opposite = radii]
	$\hat{K}_1 = 54° - 18° = 36°$	
	$T\hat{M}K = 90°$	[sum of int ∠'s of Δ]
	$\therefore$ KM = ML	[line from centre ⊥ to chord]

8.2.1	$\frac{DC}{CS}=\frac{20}{12}=\frac{5}{3}$ $\therefore \frac{DC}{CS}=\frac{RB}{BS}$ $\therefore$ BC || DR [converse line || one side of Δ OR sides in the $\therefore$ BC || AD same proportion]
8.2.2	$\frac{AR}{AD}=\frac{RB}{BS}$ [line || one side of Δ] **OR**[Prop Theorem AB || DS] $\frac{AR}{AD}=\frac{5}{3}$ $\frac{48-AD}{AD}=\frac{5}{3}$ $\therefore 5AD = 144-3AD$ $AD = 18$ $AB = 20$ [opp sides of parm] $\therefore AD : AB = 18 : 20 = 9 : 10$

OR/OF

$\frac{AR}{RD}=\frac{5}{8}$.............prop thm AB || DS

$\frac{AR}{48}=\frac{5}{8}$

$\therefore AR = 30$ and $AD = 18$

$\therefore \frac{AR}{RD}=\frac{AB}{DS}$........||| Δ's

$\therefore AB = 20$

$\therefore AB : AD = 18 : 20 = 9 : 10$

QUESTION 9.2

9.2	$E\hat{F}G = 180° - \hat{D}_1$	[opp $\angle$'s of cyclic quad]
	$\therefore E\hat{F}G = 180° - x$	
	$E\hat{F}G = 180° - \hat{G}$	[co-int $\angle$'s; EF || DG]
	$\hat{G} = x$	
	But $\hat{G} = \hat{D}_2$	[alt $\angle$'s; DH || FG]
	$\therefore \hat{D}_1 = \hat{D}_2 = x$	

QUESTION 10

10.1.1	$T\hat{P}R = 90°$	[$\angle$ in semi-circle]
	$S\hat{P}R = 90°$	[$\angle$'s on a straight line]
	$\therefore$ SR is a diameter	[converse $\angle$ in semi-circle]
	OR	
	$T\hat{K}R = 90°$	[$\angle$ in semi-circle]
	$S\hat{P}R = 90°$	[ext $\angle$ of cyclic quad]
	$\therefore$ SR is a diameter	[converse $\angle$ in semi-circle] **OR** [chord subtends a right angle]

10.1.2	$\hat{R}_1 = P\hat{T}K$ [ext $\angle$ of cyclic quad] $\hat{P}_1 = P\hat{T}K = \hat{R}_1$ [$\angle$s opp equal sides] $\hat{S} + \hat{R}_1 = \hat{P}_1 + P_2$ [ext $\angle$ of Δ] $\therefore \hat{S} = \hat{P}_2$ [$\hat{R}_1 = \hat{P}_1$]
10.1.3	In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $\Delta SPK \mathbin{\vert\vert\vert} \Delta PRK$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $S\hat{P}K = P\hat{R}K$ [sum of $\angle$s in Δ] $\Delta SPK \mathbin{\vert\vert\vert} \Delta PRK$
10.2	$\frac{PK}{RK} = \frac{SK}{PK}$ [$\Delta SPK \mathbin{\vert\vert\vert} \Delta PRK$] $PK^2 = SK.RK$ $ST^2 = SK^2 + TK^2$ [Pythagoras] $TK = PK$ [Given] $ST^2 = SK^2 + PK^2$ $ST^2 = SK^2 + SK.RK$ $ST^2 = (2RK)^2 + 2RK.RK$ $ST^2 = 6RK^2$ $ST = \sqrt{6}RK$

PAPER J

QUESTION 8.2 & 8.3

8.2	$\hat{O}_1 = 4x + 100°$	[given]
	$\therefore \hat{A} = 2x + 50°$	[$\angle$ at centre $= 2 \times \angle$ at circumference]
	$x + 34° + 2x + 50° = 180°$	[opp $\angle$s of cyclic quad]
	$3x = 96°$	
	$x = 32°$	
	OR	
	$\hat{O}_2 = 2x + 68°$	[$\angle$ at centre $= 2 \times \angle$ at circumference]
	$4x + 100° + 2x + 68° = 360°$	[$\angle$s round a pt]
	$6x = 192°$	
	$x = 32°$	
	OR	
	$\hat{O}_2 = -4x + 260°$	[$\angle$s round a pt]
	$2\hat{C} = -4x + 260°$	[$\angle$ at centre $= 2 \times \angle$ at circumference]
	$\hat{C} = -2x + 130°$	
	$x + 34° = -2x + 130°$	
	$3x = 96°$	
	$x = 32°$	

8.3.1	$O\hat{M}B = 90°$	[$\angle$ in semi circle]
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$	
	$\therefore MB = 5\sqrt{3}$	[line from centre $\perp$ to chord]
	$OB^2 = OM^2 + MB^2$	[Pythagoras]
	$OB^2 = 5^2 + \left(5\sqrt{3}\right)^2$	
	$OB = 10$ units	

QUESTION 9

9.1	$\frac{FB}{EB}=\frac{DA}{EA}$ [prop theorem; DC ∥ AB] **OR** [line ∥ one side of Δ] $FB=\frac{4p\times 21}{7p}$ FB = 12 units
9.2	In ΔEDF and ΔEAB: $\hat{E}$ is common $E\hat{D}F=\hat{A}$ [corresp ∠s; EA ∥ CB] $E\hat{F}D=E\hat{B}A$ [corresp ∠s; DC ∥ AB] ΔEDF ||| ΔEAB [∠;∠;∠]
9.3	$\frac{DF}{AB}=\frac{ED}{EA}$ [||| Δs] $DF=\frac{3p\times 14}{7p}$ DF = 6 units FC = 8 units [DC = AB = 14 units; opp sides of ∥^{m}] **OR** ΔEDF ||| ΔBCF [∠;∠;∠] $\frac{ED}{BC}=\frac{DF}{CF}$ [||| Δs] $\frac{3}{4}=\frac{14-FC}{FC}$ [BC = AD; opp sides of ∥^{m}] $3FC=56-4FC$ $FC=8$

QUESTION 10

10.1	$\hat{S}_3 = P\hat{Q}R$	[ext $\angle$ of cyclic quad]
	$\hat{R}_3 = P\hat{Q}R$	[$\angle$s opp equal sides]
	$\therefore \hat{S}_3 = \hat{R}_3$	
	But $\hat{S}_4 = \hat{R}_3$	[$\angle$s in the same seg]
	$\therefore \hat{S}_3 = \hat{S}_4$	
10.2	$\hat{R}_1 + \hat{R}_2 = P\hat{Q}R$	[tan chord theorem]
	$\hat{S}_4 = P\hat{Q}R$	[proved in 10.1]
	$\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$	
	SMRC is a cyclic quad	[converse ext $\angle$ of cyclic quad]

10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$	[ext $\angle$ of Δ]
	$\hat{S}_4 = \hat{P}_1 + \hat{A}_2$	[ext $\angle$ of Δ]
	$\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$	
	But $\hat{P}_1 = \hat{R}_2$	[tan chord theorem]
	$\therefore \hat{P}_2 = \hat{A}_2$	
	RP is a tangent to the circle	[converse tan chord theorem] **OR** [$\angle$ between line and chord] **OR** [converse alt seg theorem]
	OR	

	In ΔMSP and ΔMPA
	$\hat{M}_2$ is common
	$AR = AP$ [tans from same point]
	$\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$ [$\angle$s opp equal sides]
	$\hat{S}_4 = \hat{R}_1 + \hat{R}_2$ [proved in 10.2]
	$\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$
	$\therefore \hat{P}_2 = \hat{A}_2$ [sum of $\angle$s in Δ]
	RP is a tangent to the circle [converse tan chord theorem]